Wednesday, October 5, In-Class Exercises

1. Century-Minus-One Village is a complex of condominiums, which legally reverts back to the developer after 99 years. Assume that each unit yields its owner a constant real yearly income of $2,000 (net of repairs and maintenance fees) and is priced in the real estate market to yield a 5 percent real annual return (rental income plus price appreciation). What will the market price of a unit be at the end of 99 years, just before it reverts to the developer? After 98 years? After 97 years? What will the initial price be? Roughly sketch the market price over the first 200 years.

2. An international commercial construction company bought 300 acres in the Central Valley for $4,000 an acre in 2004. It financed the purchase with $200,000 cash and a $1,000,000 loan at 6%. The unpaid balance on this loan is currently $836,858. Now the company is deciding between two options:

a. Build a residential community with 900 homes. The company will spend $180 million the first year to build the community and will sell the homes for $225 million the second year. There will be no additional income or expenses after the second year.

b. Construct a quarry that will produce crushed rock and sand ("aggregate") used in construction projects. The quarry will cost $106 million in the first year to develop, then generate annual profits (revenue net of all wages, taxes, and other expenses) of $11.5 million the next year, growing at 3% annually for 50 years, after which the quarry site will be closed and have no commercial value.

Which option do you recommend?
1. For the first 99 years, where \( t \) is the number of years left before reversion, the value is

\[
V = \frac{2,000}{(1 + R)^t} + \frac{2,000}{(1 + R)^{t+1}} + \ldots + \frac{2,000}{(1 + R)^99} = \frac{2,000}{R} \left(1 - \frac{1}{(1 + R)^{100-t}}\right)
\]

Then, the value will be, as with a perpetuity, \( \frac{2,000}{0.05} = \$40,000 \)

As noted in the textbook, if a stock (or other asset) is priced using a required return of \( R \), the annual income plus price appreciation will give a rate of return equal to \( R \). Here, for example, the values initially \( (t = 0) \) and after 1 year \( (t = 1) \) are

\[
V_0 = \frac{2,000}{0.05} \left(1 - \frac{1}{(1 + 0.05)^{100}}\right) = \$39,695.82
\]

\[
V_1 = \frac{2,000}{0.05} \left(1 - \frac{1}{(1 + 0.05)^{99}}\right) = \$39,680.61
\]

The first-year return, income plus capital loss, is 5%:

\[
R = \frac{\frac{2,000}{39,695.82} + \frac{39,680.61-39,695.82}{39,695.82}} = 0.05038 - 0.00038 = 0.05
\]
2. Homes:

\[ V = -180 + \frac{225}{(1 + R)} \]

Quarry:

\[ V = -106 + \frac{11.5}{(1 + R)^1} + \frac{11.5(1 + g)}{(1 + R)^2} + \frac{11.5(1 + g)^2}{(1 + R)^3} + \ldots + \frac{11.5(1 + g)^{49}}{(1 + R)^{50}} \]

\[ = \frac{11.5}{R - g} \left[ 1 - \left( \frac{1 + g}{1 + R} \right)^{50} \right] \]

\[ = \frac{11.5}{R - 0.03} \left[ 1 - \left( \frac{1 + 0.03}{1 + R} \right)^{50} \right] \]

Present value, millions of dollars

Break-even 11.9%