

Final Examination Answers

1. survivor bias
2. a. H_0 : average price change = 0
 test: matched-pair t-test
- b. H_0 : average difference in daily returns is 0
 test: matched-pair t-test
- c. H_0 : chance of bradycardia for people named Brady is 0.0036
 test: 1-sample binomial test
- d. H_0 : average rating is the same for both groups
 test: difference in means t-test
- e. H_0 : average chance of COVID-19 infection is the same for both groups
 test: difference-in-proportions Z test or chi-square test
3. a. We can do a matched-pair test of the null hypothesis that the average difference is equal to zero. The steps: calculate the difference between each pair of prices; calculate the mean and standard deviation of these differences; calculate the t-value $t = \frac{\bar{X} - 0}{s / \sqrt{n}}$ and the two-sided p-value for this t-value with $20 - 1 = 19$ degrees of freedom. The answers are shown below

| | US | UK | US-UK |
|----------------------------------|-----|-----|-------|
| Big Mac | 550 | 508 | 42 |
| Quarter Pounder w/ Cheese | 520 | 518 | 2 |
| Double Quarter Oounder w/ cheese | 740 | 750 | 10 |
| Hamburger | 250 | 250 | 0 |
| Cheeseburger | 300 | 301 | -1 |
| Double Cheeseburger | 450 | 445 | 5 |
| McChicken | 400 | 388 | 12 |
| Filet-o-Fish | 380 | 329 | 51 |
| Sausage & Egg McMuffin | 480 | 430 | 50 |
| 6-piece chicken nuggets | 255 | 259 | -4 |
| French Fries (regular) | 320 | 337 | -17 |
| Pancakes | 580 | 477 | 103 |
| Apple Pie | 240 | 250 | -10 |
| McFlurry Oreo | 340 | 258 | 82 |
| Chocolate Milkshake | 520 | 468 | 52 |
| Vanilla Milkshake | 510 | 469 | 41 |
| Strawberry Milkshake | 530 | 458 | 72 |
| Cappuccino | 120 | 128 | -8 |
| Caramel Frappuccino | 420 | 399 | 21 |
| Hot Chocolate | 370 | 231 | 139 |

| | | | |
|------|----------|----------|---------|
| mean | 413.75 | 382.65 | 32.1 |
| SD | 145.5740 | 139.5890 | 42.0925 |

$$t = \frac{\bar{X} - 0}{s/\sqrt{n}} = \frac{32.1 - 0}{42.0925/\sqrt{20}} = 3.41$$

$$2p = 0.0029$$

b. The 413.75 average U.S. calories is 32.1 calories higher than the 382.65 average U.K. calories, an 8.4 percent difference. I think this is substantial, but you may disagree.

4. They are all false.

5. Using Bayes' rule, with "Epic+" meaning that ESM generated a sepsis alert and "Epic-" meaning it did not:

$$\begin{aligned}
 P[\text{no sepsis if ESM+}] &= \frac{P[\text{no sepsis}]P[\text{ESM+ if no sepsis}]}{P[\text{no sepsis}]P[\text{ESM+ if no sepsis}] + P[\text{sepsis}]P[\text{ESM+ if sepsis}]} \\
 &= \frac{\left(\frac{35,903}{38,455}\right)\left(\frac{35,903-29,775}{35,903}\right)}{\left(\frac{35,903}{38,455}\right)\left(\frac{35,903-29,775}{35,903}\right) + \left(\frac{2,552}{38,455}\right)\left(\frac{843}{2,552}\right)} \\
 &= \frac{\left(\frac{6,128}{38,455}\right)}{\left(\frac{6,128}{38,455}\right) + \left(\frac{843}{38,455}\right)} \\
 &= \frac{6,128}{6,128 + 843} \\
 &= \frac{6,128}{6,971} \\
 &= 0.88
 \end{aligned}$$

A two-way table can also be used:

| | Epic+ | Epic- | Total |
|-----------|-------|--------|--------|
| Sepsis | 843 | 1,709 | 2,552 |
| No sepsis | 6,128 | 29,775 | 35,903 |
| Total | 6,971 | 31,484 | 38,455 |

The false positive frequency is $6,128/6,971 = 0.88$

6. A chi-square test is appropriate here:

| | Actual | Expected |
|----------|--------|----------|
| rock | 27 | 40 |
| scissors | 54 | 40 |
| paper | 39 | 40 |
| Total | 120 | 120 |

$$\chi^2 = \frac{(27-40)^2}{40} + \frac{(54-40)^2}{40} + \frac{(39-40)^2}{40} = 9.15$$

The p-value is 0.0102

7. Dropping variables because of low t -values is generally a bad idea because it is likely to bias the estimated coefficients of the variables left in the model.
8. Regression to the mean, just like the tallest parents tend to be taller than their adult children and the tallest children tend to be taller than their parents.
9. These are inverse probabilities. Instead of looking at the percent of hospitalized persons who were fully vaccinated, we want to compare the fractions of the fully vaccinated and not fully vaccinated who were hospitalized.

Let V represent fully vaccinated and H represent hospitalized

$$P[H \text{ if } V] = \frac{301}{5,634,634} = 0.00005342$$

$$P[H \text{ if not } V] = \frac{515 - 301}{6,937,546 - 5,634,634} = 0.00016425$$

10. There are two ways of being dealt a blackjack: an ace followed by a king, queen, jack, or 10; or a king, queen, jack, or 10 followed by an ace, with respective probabilities $(4/52)(16/51)$ and $(16/52)(4/51)$. The total probability is $2(4/52)(16/51) = 0.0482655$
11. Using the binomial distribution with $\pi = 0.5$,

$$P[X \geq 410] = \binom{600}{410} 0.5^{410} 0.5^{600-410} + \binom{600}{411} 0.5^{411} 0.5^{600-411} + \dots + \binom{600}{600} 0.5^{600} 0.5^{600-600}$$

$$= 4.09 \times 10^{-20}$$

The two-sided p -value is 8.18×10^{-20} .

12. Each estimated coefficient is *ceteris paribus*, holding the other explanatory variables constant.
13. These statements are all false. (Adapted from Gigerenzer, G. (2004), "Mindless Statistics," *Journal of Socio-Economics*, 33, 587–606.)
14. (1) With 4 seasons and 20 years, there are 80 observations, not 20; (2) R^2 is not a percent but a number between 0 and 1; (3) p -values cannot be negative; (4) for $t = 2.62$, the p -value is much lower than 0.9894; and (5) with 4 seasonal categories, there should be only 3 dummy variables.

15. a. $p = (1/2)^{10} = 1/1024$
- b. $w = 1/p = 2^{10} = 1024$

$$c. 1 - \text{probability that it never happens} = 1 - (1 - p)^w = 1 - \left(1 - \left(\frac{1}{2}\right)^{10}\right)^{2^{10}} = 1 - \left(\frac{1023}{1024}\right)^{1024} = 0.6323$$

In general:

- a. $p = (1/2)^m$
- b. $w = 1/p = 1/(1/2)^m = 2^m$

$$c. \quad 1 - (1-p)^w = 1 - \left(1 - \frac{1}{2^m}\right)^{2^m}$$

$$\text{If } m = 1 \text{ then } p = 1/2, w = 2, \text{ and } 1 - (1-p)^w = 1 - \left(1 - \frac{1}{2}\right)^2 = 0.75$$

As m increases, so does $x = 2^m$ and

$$\frac{\partial \left(1 - \left(1 - \frac{1}{x}\right)^x\right)}{\partial x} = \left(1 - \frac{1}{x}\right)^x \ln\left(1 - \frac{1}{x}\right) \frac{1}{x^2} < 0$$

In the limit,

$$\lim_{m \rightarrow \infty} 1 - \left(1 - \frac{1}{2^m}\right)^{2^m} \rightarrow 1 - \frac{1}{e} = 1 - 0.367879 = 0.632121$$

so the answer to Part c is always > 0.632121

16. This is a multiplication problem. The probability that the first tile will not be a bonus tile is $136/144$. The probability that the second tile will not be a bonus tile, given that the first tile picked was not a bonus tile is $135/143$. Thus the probability that the first two tiles are not bonus tiles is $(136/144)(135/143)$. The probability of picking $144 - 16 = 128$ tiles without any being a bonus tile is

$$\frac{136}{144} \frac{135}{143} \frac{134}{142} \cdots \frac{9}{17}$$

17. First, we want to compare these stocks to other stocks, not to the S&P 500 which includes these stocks. Second, we should identify the five biggest stocks in 2009 and see how they did subsequently. The biggest stocks today may be the ones that performed the best, rather than vice versa.
18. Walker left the 5-hour category out of his graph, evidently because it contradicted his argument.
19. The table shows that there was indeed a 24 percent increase after the spring change and a 21 percent decrease after fall change, as Walker stated, but these fluctuations did not happen, as Walker claimed, the “following day.” Saturday night is when we have one less or one more hour of sleep and the following day is Sunday. Instead, the spring increase in the Michigan data was on a Monday and the fall decrease was on a Tuesday, two seemingly random days—except for the fact that they were the only days to have had p-values (0.011 and 0.044, respectively) below 0.05. On the Sunday after the time changes, the number of heart attacks went in the opposite direction.
20. The two lines would have to start at the same point (for example, 100) in order to see which increased more:

