

Final Examination Answers

1. This is a *post hoc ergo propter hoc* fallacy. More than 100 million Americans had been vaccinated and the overall annual number of deaths in the United States is between 8 and 9 people per thousand. Applying that death rate to a month and 100 million people, we would expect about 7,000 deaths. For a more precise estimate we should omit deaths from automobile accidents and the like, but also take into account that the elderly were the first to be vaccinated. According to the CDC website,

CDC and FDA physicians review each case report of death as soon as notified and CDC requests medical records to further assess reports. A review of available clinical information, including death certificates, autopsy, and medical records has not established a causal link to COVID-19 vaccines.

2. a. The average ages are equal; two-sample t-test
b. Hospitalization and physical activity are independent; chi-square
c. The probability that an infected person is female is 0.5; one-sample proportions test
d. Age and physical activity are independent; chi-square
e. There is no effect of activity on chances of dying; multiple regression
3. a. A difference-in-means test does not have to assume that the two samples have the same standard deviation. A matched-pair test does require genuinely matched pairs.
b. We don't test alternative hypotheses.
4. We can do a multiple regression model with daily sales as the dependent variable and six dummy variables for the seven days of the week. (A chi-square test is not appropriate because the value of sales are not count data.)

5. a. The sample success proportion is $14/29 = 0.483$, which seems substantially higher than 0.396 to me. Using an exact binomial test,

$$P[X \geq 14] = \binom{29}{14} 0.396^{14} (1-0.396)^{29-14} + \binom{29}{15} 0.396^{15} (1-0.396)^{29-15} + \dots + \binom{29}{29} 0.396^{29} (1-0.396)^{29-29} \\ = 0.2206$$

for a two-sided p-value of $2(0.2206) = 0.4412$

- b. If the sample had been 10 times larger with the same success proportion, the question of substantial would be the same, but the p-value for statistical significance would be lower:

$$P[X \geq 140] = \binom{290}{140} 0.396^{140} (1-0.396)^{290-140} + \binom{290}{141} 0.396^{141} (1-0.396)^{290-141} + \dots + \binom{290}{290} 0.396^{290} (1-0.396)^{290-290} \\ = 0.0017$$

for a two-sided p-value of $2(0.0017) = 0.0034$

6. a. There are five ways to roll a 6 (1-5, 2-4, 3-3, 4-2, and 5-1) and five ways to roll an 8 (2-6, 3-5, 4-4, 5-3, and 6-2), so each has a probability $5/36$.
b. In a small number of rolls, it is unlikely that all of the observed frequencies will be close to the theoretical probabilities.
c. When something has been happening far less often than expected, it is (by definition) unlikely to continue happening so infrequently. On average, it will come up as often as expected—which is more

frequently than it has been coming up.

7. The t-value, $19.350/20.945$, is less than 2; however, this does not prove that there is no effect.
8. This sure sounds like data mining, which is good at identifying past patterns that are unreliable guides to the future. A horse named Justify ended up winning the Kentucky Derby and the Triple Crown that year.
9. Regression toward the mean
10. This is a Bayes' Rule problem. Let G represent having a glitch and let T represent a ticket being generated:

$$\begin{aligned}P[G \text{ if } T] &= \frac{P[G]P[T \text{ if } G]}{P[G]P[T \text{ if } G] + P[\text{not } G]P[T \text{ if not } G]} \\&= \frac{(0.90)(0.60)}{(0.90)(0.60) + (0.10)(0.20)} \\&= \frac{0.54}{0.56} = 0.9642\end{aligned}$$

11. The probability that no one in a group of 25 will test positive is equal to 0.99^{25} . Therefore, the expected value of the number of tests for a group of 25 is

$$1(0.99^{25}) + (1+25)(1 - 0.99^{25}) = 1 + 25(1 - 0.99^{25}) = 6.55$$

12. The null hypothesis concerns the unknown probability π that a randomly selected player will be born during the first six months of the year, not the known fraction p . The Z -value should compare the observed fraction p to the null hypothesis value 0.5, and the entire denominator is inside the square root sign and uses the null hypothesis value 0.5, not p :

$$Z = \frac{p - 0.5}{\sqrt{0.5(1 - 0.5)/n}}$$

13. There are 16 wind tiles and $144 - 16 = 128$ non-wind tiles
 - a. $(128/144)(127/143) \dots (113/129)$
 - b. $(128/144)(127/143) \dots (65/81)$
 - c. $(16/144)(15/143) \dots (1/129)$
 - d. $(16/144)(12/143)(8/142)(4/141)$
14. a. No, the central limit theorem implies that the sample mean approaches a normal distribution even if the individual scores are not normally distributed.
 - b. $X_2 + X_3 = 1$, so one of these explanatory variables is redundant and can be omitted. Logically, we cannot estimate the effect of either variable on Y , holding the other variable constant.
15. Statement b is true; a , c , and d are false.

$$16. \mu = \left(\frac{1}{3}\right)(1) + \left(\frac{2}{3}\right)\left(-\frac{1}{4}\right) = \frac{1}{6}$$

17. It would be better to use actual sale prices instead of list prices. The dependent variable should be the price,

not the square footage, and there should be additional explanatory variable like the lot size, number of bathrooms, and whether the home has a swimming pool. There seems to be an error in the labeling of the horizontal axis, in that it is unlikely that the prices were in British pounds. It would be helpful to show the p -values for the coefficients of the explanatory variables. I don't know what "line of least regression" means.

18. Since every observed value is larger than the corresponding expected value, the sum of the observed values is not equal to the sum of the expected values.
19. a. 7.48 is the actual average score of these 100 students, not the expected value of an average score.
b. Even if the null hypothesis is rejected, we don't know for certain that it is incorrect. Here, with a 5% level of significance, there is a 5% chance of incorrectly rejecting the null hypothesis that the mean is 50. With a two-tail test, we can see whether we are rejecting the null hypothesis because the sample mean is less than or greater than 50 (though, again, we may be incorrectly rejecting the null hypothesis).
20. The dates are suspicious. It is unlikely that March 19, 2020, was the only day that Motley Fool recommended Zoom. Why, on January 5, 2021, are they talking about performance as of October 12, 2020, three month's earlier? The chart below confirms that Zoom did not do well between October 12, 2020, and January 5, 2021.

In addition, we don't know how strongly they recommended Zoom, or the performance of the other stocks that they recommended at that time. Perhaps they recommended 25 stocks equally strongly and only one (Zoom) did well.

