

Final Examination (150 minutes)

No calculators allowed. Just set up your answers, for example, $P = 49/52$. BE SURE TO EXPLAIN YOUR REASONING. If you want extra time, you can buy time at a price of 1 point a minute; for example, if your test is handed in 10 minutes after the scheduled finish time, 10 points will be subtracted from the test score.

1. A *New York Times* article reported that

Money may not buy happiness, but the Fed survey suggests it buys good health. About 90 percent of the 1 percenters [top 1% of the wealth distribution] describe themselves as being in excellent or good health, compared with 75 percent of everybody else. About 85 percent expect to live into their 80s, compared with 68 percent of everybody else.

As a statistician, what do you say?

2. Stanford's alcohol policy

prohibits containers 750 mL [the size of a standard wine bottle] and larger of distilled liquor, spirits and hard alcohol (alcohol by volume 20 percent and above or 40 proof) in undergraduate student residences, including rooms and common spaces....Limiting the size of hard alcohol containers is a harm reduction strategy designed to reduce the amount of high-volume alcohol content that is available for consumption at a given time. We feel it is a sensible, creative solution that has roots in research-based solutions.

As a statistician, what do you say?

3. A taxi-cab company was involved in a hit-and-run accident at night. Two cab companies, Green and Blue, operate in the city, with 85% of the cabs Green and 15% Blue. A witness identified the cab as Blue. The court tested the reliability of the witness under the circumstances that existed on the night of the accident and concluded that the witness correctly identified a cab's color 80% of the time and failed 20% of the time. What is the probability that the cab involved in the accident was Blue?

4. The Scholastic Aptitude Test used to have a wrong-answer penalty. If a multiple-choice question has 5 possible answers, a student was given 1 point for a correct answer and a 1/4-point deduction for a wrong answer. What was the expected value of answering a question with a completely random guess? Now there is no wrong-answer penalty. What is the expected value of a random guess?
5. Alex and Blaine are having a contest shooting basketball free throws, where the first person to make a basket wins. Alex has a 2/3 chance of making a basket; Blaine has a 1/3 chance. Assuming independence, what is the probability that Blaine will win if Blaine shoots first?

6. Identify the most serious problem with this model of household income

$$Y = \alpha + \beta_1 D + \beta_2 E + \beta_3 A + \varepsilon$$

where Y = average household income in state

D = state dummy variable, = 1 for Alabama, 2 for Alaska, 3 for Arizona, etc.

E = average education in state

A = average age in state

7. Data from a random sample of 30 first-year college students who lived in single rooms and 30 who lived in doubles were used to estimate this equation.

$$Y = 2.31 - 0.027G + 0.144H + 0.045R, \quad R^2 = 0.243$$

[0.439] [3.036] [0.716]

where Y = grade point average (4-point scale); G = 1 if male, 0 if female; H = happiness (6-point scale), R = 1 if have roommate, 0 if not, and the t-values are in brackets. Explain the error in this interpretation of the results: "I expected that students who did not have roommates would score low on the happiness scale and consequently have low GPAs. However, the coefficient of the roommate dummy variable is not close to being statistically significant."

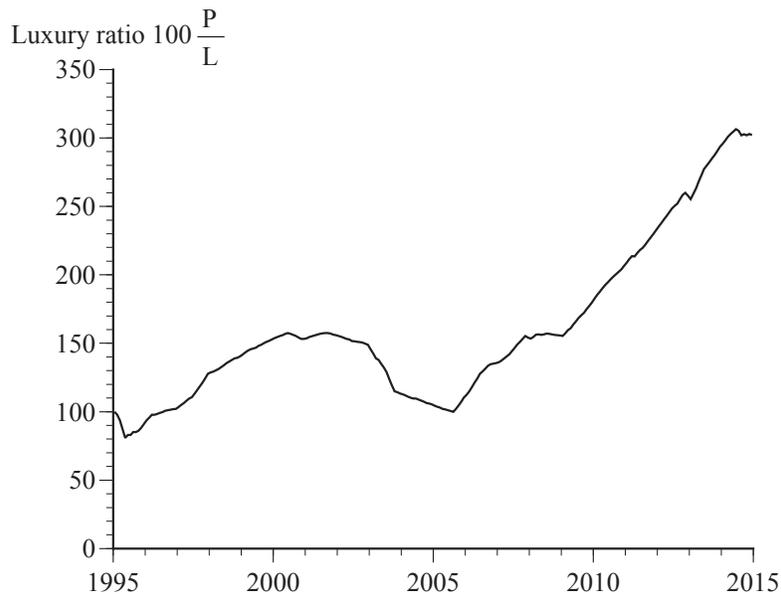
8. A finance PhD student analyzed monthly data during the years 1995 to 2014 for two home price indexes, one (“P”) for homes in the prime London neighborhoods of Kensington and Chelsea, and the City of Westminster and the other (“L”) for homes in Liverpool. The home price indexes were both scaled to equal 100 in January 1995. Her descriptive statistics were

	Liverpool Index L	Prime Index P
Mean	145.7	339.0
Standard Deviation	26.6	138.9
Minimum	100.0	100.0
Maximum	191.4	578.0

She also calculated the luxury ratio each year

$$\text{Luxury ratio} = 100 \frac{P}{L}$$

and constructed this graph:



Critically evaluate each of these conclusions:

- “The standard deviation is larger for prime properties because of the greater diversity of more expensive homes in prime London neighborhoods as compared with the Liverpool housing market.”
- “The luxury ratio of prime London with Liverpool represents the average excess amount, in percentage terms, that is paid in prime London relative to Liverpool for a property. A luxury ratio of 150, for example, signifies that the price level of prime property is 50% greater than the price in Liverpool.”
- “We can also observe that 2005 was the right moment to buy prime property in London if one meant to do so. Premiums paid in prime London with respect to Liverpool greatly increased after 2005.”

9. A student estimated a model of airline prices based on trip distances:

$$P = 138.97 + 0.062M - 47.83A + 0.021M*A$$

where P is the ticket price, M is the miles the flight traveled, A = 1 if it is an American Airlines flight, and = 0 if it is a Delta Airlines flight. He also estimated a separate equation for American Airlines

$$P = 91.14 + 0.083M$$

and e-mailed me this question:

You mentioned after class that the values should be the same as when I did a simple regression of each airline separately, but I didn't get any of the same values. I'm not sure if I misheard you or I did something wrong in my analysis?

How would you respond?

10. A professor wanted to estimate this model with ordinary least squares using cross-section data:

$$C = \alpha + \beta_1 Y + \beta_2 Y(-1) + \beta_3 (Y - Y(-1)) + \beta_4 W + \varepsilon$$

where C is the household's spending in 2015, Y is the household's income in 2015, Y(-1) is the household's income in 2014, Y - Y(-1) is the change in this household's income between 2014 and 2015, and W is the household's wealth at the beginning of 2015. Interpret each of the beta coefficients. What problem do you see with this model?

11. Answer this recent Car Talk Puzzler:

There are two rookie baseball players, Bluto and Popeye, who started the season on opening day and made a wager as to which one would have the best batting average at the end of the season. Well, the last day of the season arrives, and not much is going to change--especially considering that neither one of them is in the starting lineup.

Bluto says, "Hey, Popeye, what did you bat for the first half of the year?" Popeye answers, "I batted .250." And Bluto responds, "Well, I got you there. I batted .300. How about after the All-Star break?" Proudly, Popeye pipes up, "I batted .375." Bluto says, "Pretty good, but I batted .400. Fork over the 20 bucks that we bet."

The bat boy, Dougie, saunters over and says, "Don't pay the 20 bucks, Popeye. I think you won." The question is this: How could Popeye have won?

(A .250 batting average means Popeye got hits in 25% of his at-bats; for example, 5 of 20 is $5/20 = 0.250$.)

12. Identify the most appropriate statistical test for each of these studies (for example, two-sample t-test, ANOVA, or multiple regression):
- A study compared the success rates of soccer penalty shots taken to the left, right, and middle of the goal.
 - A study compared male and female ratings (on a scale of 1 to 10) of Coke, Diet Coke, and Caffeine-Free Coke.
 - A study of the effect of quarries on home prices compared the prices of homes located different distances from a quarry, taking into account each home's square footage, lot size, number of bedrooms, number of bath rooms, and age of the home.
 - A study compared the income of immigrant mothers and their daughters when each daughter was the same age as her mother at the time of the daughter's birth (for example, if the daughter was born when her mother was 28, the study compared the mother and the daughter's income when each was 28).
 - A study compared daily stock-market returns on the New York Stock Exchange to the amount of cloudiness that day in New York (measured on a scale from 0 to 100).
13. A 2016 *Sports Illustrated* article noted that the Chicago Cubs had won 24 of their first 30 Major League Baseball games, and that only 13 teams had done so well in the modern baseball era. As a statistician, what pattern(s) do you see in these data and how would you explain them?

	First 30 Games			Rest of Season			Total Season		
	Wins	Losses	Percent	Wins	Losses	Percent	Wins	Losses	Percent
1905 Giants	24	6	80.0	81	42	65.9	105	48	68.6
1907 Cubs	24	6	80.0	83	39	68.0	107	45	70.4
1907 Giants	25	5	83.3	57	66	46.3	82	71	53.6
1911 Tigers	25	5	83.3	64	60	51.6	89	65	57.8
1921 Pirates	24	6	80.0	66	57	53.7	90	63	58.8
1928 Yankees	24	6	80.0	77	47	62.1	101	53	65.6
1939 Yankees	24	6	80.0	82	39	67.8	106	45	70.2
1946 Red Sox	24	6	80.0	80	44	64.5	104	50	67.5
1955 Dodgers	25	5	83.3	73	50	59.3	98	55	64.1
1958 Yankees	24	6	80.0	68	56	54.8	92	62	59.7
1977 Dodgers	24	6	80.0	74	58	56.1	98	64	60.5
1981 A's	24	6	80.0	40	39	50.6	64	45	58.7
1984 Tigers	26	4	86.7	78	54	59.1	104	58	64.2

14. A computer program was written to divide a class of 21 students randomly into seven 3-person teams. If there are 5 females in the class, what is the probability that the first 3-person team chosen will be all female?

15. A professor estimated a model that correctly predicts the outcomes of the last 10 presidential elections, with an R^2 of 1.000:

$$i\% = 78.18 - 7.36iP - 27.20iS + 14.88cS - 34.46iG + 8.33cG - 19.54iR + 3.49cR - 13.07iVP + 7.93cVP$$

where

$i\%$ = Percent of major party vote received by the incumbent party's candidate

iP = 1 if the incumbent party candidate is President, 0 otherwise

iS = 1 if the incumbent party candidate has been a U.S. Senator, 0 otherwise

cS = 1 if the challenger party candidate has been a U.S. Senator, 0 otherwise

iG = 1 if the incumbent party candidate has been a state governor, 0 otherwise

cG = 1 if the challenger party candidate has been a state governor, 0 otherwise

iR = 1 if the incumbent party candidate has been a U.S. Representative, 0 otherwise

cR = 1 if the challenger party candidate has been a U.S. Representative, 0 otherwise

iVP = 1 if the incumbent party candidate has been a U.S. Vice President, 0 otherwise

cVP = 1 if the challenger party candidate has been a U.S. Vice President, 0 otherwise

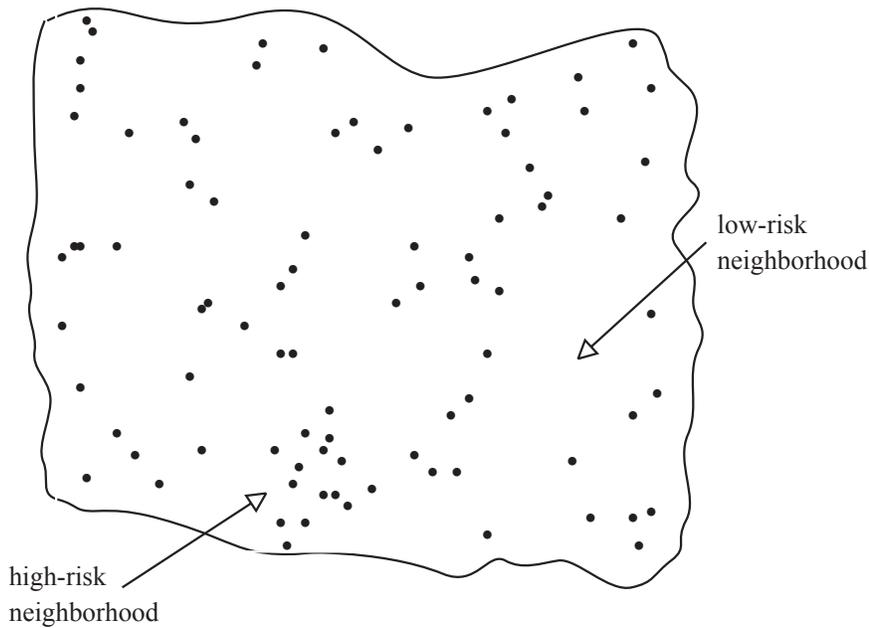
For example, this model's prediction of Hillary Clinton's share of the 2016 two-party vote is exactly equal to the actual 50.98% share she received:

$$\begin{aligned} i\% &= 78.18 - 7.36(0) - 27.20(1) + 14.88(0) - 34.46(0) + 8.33(0) - 19.54(0) + 3.49(0) - 13.07(0) + 7.93(0) \\ &= 50.98 \end{aligned}$$

What problem do you see with this model?

16. Professor Smith wants to select 20 of the 21 students in his class for ten 2-person teams; the student who is not selected does not have to do the assignment. Smith writes the numbers 1 through 20 on slips of paper, and places these 20 slips of paper in a hat, along with a blank piece of paper, and lets each student pick a slip of paper out of the hat. Whoever chooses the blank slip is given an A and does not have to do the assignment. The students who choose numbers 1 and 2 are on the same team, as are the students who choose numbers 3 and 4, and so on. If you are one of these 21 students and want to draw the blank paper, would you rather be the first or last student to pick a slip of paper out of the hat?

17. A statistician looked at mortality data for a county in Ohio and identified the home addresses of people who had died of cancer within the past 50 years, represented by dots in this county map:



The statistician found a high-risk neighborhood and a low-risk neighborhood. She drove through these neighborhoods and found that there were many more live-oak trees in the high-risk neighborhood than in the low-risk neighborhood. When she did a difference-in-proportions test comparing the cancer frequencies in these two neighborhoods, she got a Z-value of 3.89 and a two-sided p-value of 0.0001. A peer-reviewed journal published her results, including her recommendation that cities could reduce cancer risks by removing live-oak trees.

How would you respond? Make a clear argument that might persuade a city council.

18. Two professors looked at the birth months of 76 major league baseball players who committed suicide. They calculated the adjusted number of suicides in each birth month by dividing the number of suicides in that month by the total number of players with that birth month and multiplying by 1000. For example, there were a total of 638 players with January birth months; so, the adjusted January number is $1000(6/638) = 9.4$, rounded off to 9. The expected value in each month is $126/12 = 10.5$ and their calculated chi-square value is

$$\chi^2 = \frac{(9-10.5)^2}{10.5} + \frac{(13-10.5)^2}{10.5} + \dots + \frac{(10-10.5)^2}{10.5} = 43.1$$

	Actual	Adjusted	Expected
January	6	9	10.5
February	7	13	10.5
March	5	8	10.5
April	5	10	10.5
May	5	9	10.5
June	6	11	10.5
July	2	3	10.5
August	19	29	10.5
September	5	8	10.5
October	7	11	10.5
November	3	5	10.5
December	6	10	10.5
Total	76	126	126

- What is most serious error with their statistical procedure?
- Do you think that their mistake increased or decreased the chi-square value?
- How would you calculate the correct chi-square value?

19. A well-known automobile manufacturer claimed in a 1986 television commercial that 70 percent of its cars that had been registered in the United States since 1974 were still on the road. Does this statistic imply that there is a 70 percent chance that one of this company's cars will last at least twelve years?

20. Explain why you either agree or disagree with this argument in Leo Gould's book, *You Bet Your Life*:
There are many systems of all kinds but in talking to gamblers you will find the one they believe in most (and it can be used for most gambling games) is the 'watch' or 'patience' system. It's simple. If you are playing dice and certain bets pay even money, before you put your first [bet] on any of the numbers watch until one shows up at least three of four consecutive times and then put your first [bet] on the opposite chance.