

Final Examination (150 minutes)

No calculators allowed. Just set up your answers, for example, $P = 49/52$. BE SURE TO EXPLAIN YOUR REASONING. If you want extra time, you can buy time at a price of 1 point a minute; for example, if your test is handed in 10 minutes after the scheduled finish time, 10 points will be subtracted from the test score.

1. American Express and the French tourist office sponsored a survey that found that most visitors to France do not consider the French to be especially unfriendly. The sample consisted of “1,000 Americans who have visited France more than once for pleasure over the past two years.” Why is this survey biased?

2. In a class of 15 students, 6 students will be chosen randomly every week to do a project. There are 10 weeks in the term. Cory is one of the students in this class.
 - a. What is the probability that Cory will be chosen every week?

 - b. What is the probability that Cory will never be chosen?

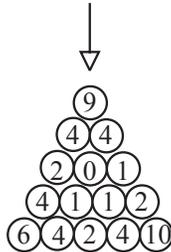
3. A survey of 7,000 college students from nearly 1,000 colleges and universities found that students, on average, expect to earn \$60,000 in their first job out of college. The actual average starting salary for these 7,000 students was \$48,000. Do not try to do the calculations; do explain what you need to calculate:
 - a. a 95% confidence interval for the average starting salary of all college students.

 - b. a test of whether the expectations are, on average, correct.

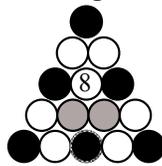
4. What statistical phenomenon would you use to explain the fact that most movie sequels are not as good as the original?

5. In the 1991 National Basketball Association (NBA) 3-point shooting contest, Craig Hodges made 21 of 25 shots in one round, including an astonishing 19 in a row. It has been estimated that, for a player of his ability, the probability of such a streak is 0.0000477. Since its beginning in 1986, 124 players have participated in the contest. Assuming that performances are independent, what is the probability that at least 1 out of 124 players will have a performance that has a 0.0000477 probability of occurring?

6. Ninety-nine break shots on a full rack of billiard balls resulted in 54 sunk balls, allocated among the 15 balls as follows (for example, the balls in the three corners were sunk 9, 6, and 10 times):



The diagram below shows how the rack was arranged before each break, with the 8 ball surrounded by seven striped balls (white in the diagram) and the seven solid balls (black). The two gray balls indicate that the seventh striped ball and seventh solid ball can be placed in either position.



Omit the 8-ball from your analysis and assume that of the two balls sunk from the gray positions, 1 was solid and 1 was striped. Is there a statistically significant relationship between a ball being solid or striped and its likelihood of being sunk on the break?

7. An investor's utility U is a function of her percentage rate of return R :

$$U = 2.5R - 0.01R^2, \quad R \leq 100$$

- a. What is the expected value of utility if R is certain to be 0?

- b. What is the expected value of utility if there are two equally likely possible values of R : $R = +50$ and $R = -50$?

8. A study of the income of family-law lawyers looked at the influence of age, gender, and law school attended:

$$Y = \alpha + \beta_1 X + \beta_2 D_1 + \beta_3 D_2 + \varepsilon$$

where Y = average income over the past 5 years, X = current age; $D_1 = 0$ if male, 1 if female; and $D_2 = 1$ if attended Harvard, $D_2 = 2$ if attended Yale, $D_2 = 3$ if attended Stanford, and $D_2 = 0$ otherwise. What is the most important problem with this model specification?

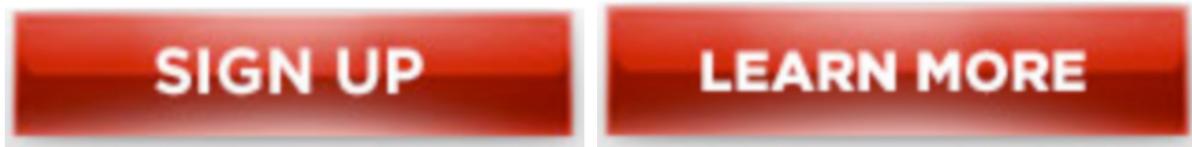
9. Answer this Car Talk puzzler:

Many years ago a prisoner was condemned to die. He was in his cell, and the warden came to visit him. He said to the prisoner that his odds of dying the next day are 100% (big damn surprise, right?) But the warden wants to increase the prisoner's chances to 50%. The warden gives the prisoner 2 shoeboxes, one with 50 white marbles, and one with 50 black marbles. The warden will come in blindfolded and pick a marble out of one of the boxes. White, he lives. Black, he dies.

The marbles can be arranged in any way. All the marbles must be used in either box. Is there any way to improve his chances above 50%? If so, what is the new probability?

10. An A/B test is like an internet laboratory experiment. Two versions of a web page are created—typically the current page and a proposed modification, perhaps with a different banner, headline, product description, testimonial, or button label. A random number generator is used to determine which page a user sees. A pre-specified metric, like purchases or mailing-list signups—is used to determine the winning page.

The Obama 2008 presidential campaign compared the number of web-page visitors who signed up for more information with these two buttons



Button	Visitors	Signups
Sign up	77,858	5,851
Learn More	77,729	6,927

- Why are the number of visitors not equal?
- Is the difference between the observed number of signups statistically significant at the 5% level?

11. Two chess players, A and B, are going to play a 3-of-5 match—the first person to win 3 games is the winner. After someone has won 3 games, the match is over. If A has a 0.60 probability of winning any game, and the outcomes are independent, what is the probability that A wins the match?

12. You have two fair coins and one coin that has heads on both sides. You pick a coin at random and toss it twice. If it lands heads on both tosses, what is the probability that it will also land heads on a third toss?

13. Answer this Car Talk puzzler:

The preferred footwear in Townberg is combat boots. Twenty percent of the people have one foot, so they wear one boot. Of the remaining people, half go barefoot and half wear two boots—and there are 20 thousand boots worn in Townberg. What's the population of Townberg?

14. Horizons ETFs Management Canada Inc. is recalibrating its artificial intelligence (AI) exchange-traded fund (ETF) after a year of underperformance that was a “complete disappointment,” according to the provider’s chief executive officer.

The Horizons Active A.I. Global Equity ETF (MIND), which uses a proprietary AI-directed selection process to invest in major global equity indexes through a basket of ETFs, has trailed the market over the past year, down 1.04 per cent on a return basis, versus a positive total return, in Canadian dollar terms, of 5.76 per cent for the MSCI World Index....

“To the extent that they may have underperformed today, there is a strong possibility that they can outperform in the future,” Mr. Hawkins [Horizons CEO] said.

Explain Hawkins’ reasoning and why you either agree or disagree with it.

15. A study found that, controlling for the number of cars and miles driven, people driving sports cars were more likely to get ticketed than were people driving minivans. Can you think of a reasonable explanation other than the police like to pick on people driving sports cars?

16. A researcher tests for precognition by asking a volunteer to predict whether the next color shown on a computer screen will be red or green. The software uses a random number generator with a 0.5 probability of showing green and a 0.5 probability of showing red. In the experimental test, the volunteer got 62 out of 100 correct. Calculate the exact p -value for a test of the natural null hypothesis.
17. Identify the most appropriate statistical test for each of these research hypotheses. You do not need to show any formulas, just identify the test, for example, “difference-in-means t test.”
- A study of the effect of clever ticker symbols on shareholder returns compared the daily returns on a portfolio of clever ticker stocks to the daily returns on the S&P 500 index of stock prices over the period 2000 through 2018.
 - A study of the effect of gender and height on college GPA collected data for 60 senior Pomona College economics majors.
 - A study of the effect of Donald Trump’s tweets on the stock market compared the number of Trump tweets sent each day with the change in the Dow Jones Industrial Average the next day.
 - A survey of 100 Pomona College students compared the percentage who were first-born or only children to the national percentage of 40%.
 - A survey of 150 Pomona College students compared the number of first-generation, second-generation, and other college attendees with their choice of a STEM major or a non-STEM major.
18. This multiple regression equation was estimated using data for 100 homes Plano, Texas:
- $$\hat{Y} = 15,034 + 104.96X_1 - 12,018X_2 + 56.68X_3 + 23,835X_4 - 5,225X_5 + 3,396X_6 \quad . \quad R^2 = 0.774$$
- (2,206) (11.70) (793) (13.70) (4,793) (2,187) (1,235)
- where Y = home price, X_1 = square feet of living area; X_2 = age of house in years; X_3 = garage square feet; X_4 = 1 if has central air, 0 if doesn’t; X_5 = number of bedrooms; X_6 = number of bathrooms; and the standard errors are in parentheses.
- Which of the estimated coefficients are statistically significant at the 5 percent level?
 - Provide a logical explanation for why the coefficient of X_5 might be negative.
 - Do you think that the parameters of housing regression equations change over time? Why or why not?

19. A study of the number of times that Trump tweeted the word *economy* each day and the daily high temperature in Moscow five days later found this relationship:

$$Y = 49.49 + 2.00Z \quad R^2 = 0.01$$

[65.13] [2.62]

where Y is the Moscow temperature ($^{\circ}\text{F}$), Z is the daily Z-score for Trump tweets, and the t-values are in brackets.

- What is the average value of Z ?
- What is the average value of Y ?
- Does the coefficient of Z seem statistically significant to you?
- Does the coefficient of Z seem substantial to you?
- Why are you deeply skeptical of this equation?

20. Explain these phenomena:

- The National UFO Reporting Center reports that there have been more reported UFO sightings in California than in any other state, indeed more than New York, Texas, and Massachusetts put together. This shows that Aliens like California.
- The National UFO Reporting Center reports that there have been more reported U.S. UFO sightings on the week of July 4th than on any other week of the year. This shows that Aliens like to watch fireworks as much as Americans do.

