

Final Examination (150 minutes)

No calculators allowed. Just set up your answers, for example,  $P = 49/52$ . If you want extra time, you can buy time at a price of 1 point a minute; for example, if your test is handed in 10 minutes after the scheduled finish time, 10 points will be subtracted from the test score.

1. In April 2021, after COVID-19 vaccines were approved for public use, Wisconsin Senator Ron Johnson said that he was “highly suspicious” of the government’s vaccination campaign and that there was “no reason to be pushing vaccines on people.” In May 2021 he argued that the vaccines may not be safe: “We are over 3,000 deaths within 30 days of getting the vaccine.” Why is his evidence statistically unpersuasive?

2. Identify the most appropriate null hypothesis and statistical test for each of these studies (for example,

$H_0$ : the average difference is zero

test: matched-pair t-test:

a. Data for 48,440 adult patients infected with COVID-19 were used to compare the average age of those who died with the average age of those who did not die.

$H_0$ :

test:

b. The infected patients were divided into three physical activity groups (consistently inactive, some activity, and consistently active) and the researchers compared the frequency with which people in each physical activity group were hospitalized.

$H_0$ :

test:

c. The number of infected male and female patients were compared to see if each sex was equally likely to be infected.

$H_0$ :

test:

d. The infected patients were divided into four age groups (< 60, 60-69, 70-79, and > 79) to see if there was a relationship between age and physical activity.

$H_0$ :

test:

e. Each patient’s probability of dying was predicted based on the patient’s physical activity category and 25 other possible confounding variables.

$H_0$ :

test:

3. Explain the errors in these remarks:
  - a. “We used a matched-pair test instead of a difference-in-means test because a matched-pair test does not assume that the two samples have the same standard deviation.”
  
  
  
  
  
  
  
  
  
  
  - b. “In the regression analysis of the relationship between commute time to work  $Y$  and number of children  $X$ , the ordinary least squares estimators will be used to test the null hypothesis and alternative hypothesis. A test of the null hypothesis will show whether there is no relationship between number of children and an individual’s commute time; a test of the alternative hypothesis will show whether there is a relationship between number of children and an individual’s commute time.”
  
  
  
  
  
  
  
  
  
  
4. The dollar value of daily sales were recorded at an on-campus college snack bar during the spring 2017 semester. How would you test whether the dollar value of sales varied by day of the week?
  
  
  
  
  
  
  
  
  
  
5. Nationally, 39.6 percent of all Americans are either first-born or only children. A random-sample of 29 students at an elite liberal arts college found that 14 were either first-born or only children.
  - a. Is this difference from the national average substantial and statistically significant at the 5% level?
  
  
  
  
  
  
  
  
  
  
  - b. How would your answers to the two questions in Part (a) change if the survey had consisted of 290 students of whom 140 were either first-born or only children? Use logic, not equations.

6. (True story) In the board game Settlers of Catan, a pair of standard 6-sided dice are rolled on each turn to determine how new resources are distributed to each player. Whenever Jim plays Settlers, he meticulously records every dice roll and inevitably finds that some numbers came up more often than expected, while other numbers come up less often than expected. For example, the numbers 6 and 8 are equally likely, but they almost never come up equally often.
- Explain why the numbers 6 and 8 are equally likely.
  - Explain why you either are or are not surprised that the numbers 6 and 8 seldom come up equally often.
  - When a number that is advantageous for Jim (say, 8) comes up far less often than expected, he switches to a different pair of dice, Explain why you think that, going forward, 8 will or will not come up more frequently than it came up with the previous dice?

7. U.S. taxpayers can either take a standard deduction or, if it is larger, an itemized deduction based on property taxes, mortgage interest, charitable contributions, and a few other items. The deduction of property taxes and mortgage interest is intended to encourage home ownership which is thought to boost active citizenship, stable neighborhoods, and other externalities. A cross-section regression equation was estimated using 2015-2019 average values for each of the 58 counties in California:

$$(\text{VotingPercentage}) = \alpha + \beta(\text{OwnerOccupiedHousingRate}) + \varepsilon$$

where *VotingPercentage* is the percentage of the eligible population who voted in the 2019 presidential primary election and *OwnerOccupiedHousingRate* is the average value during the years 2015-2019 of the ratio of the number of homeowners divided by the total number of households in each county.

The estimated value of  $\beta$  was 19.350 with a standard error of 20.945. Explain why you either agree or disagree with the authors' conclusion that,

*Although homeownership seems to have an economically significant and positive impact on voting percentage, statistical significance is lacking. Therefore, critics of the mortgage interest deduction have further support that home ownership does not encourage voting.*

8. A few days before the 2018 Kentucky Derby horserace, “the most exciting two minutes in sports,” it was reported that,

*Horses with the first initial “A” have won the race 11 times in 96 races, including Always Dreaming’s win last year. The 11.5 percent success rate is higher than any other initial, good news for fans of Audible [one of the horses running in the 2018 race].*

As a statistician, how would you respond?

9. The John Bates Clark Medal is awarded annually to the top American economist under the age of 40. The 2021 medal was given to Isaiah Andrews. One of his interests is the “winner’s curse,” which was described by *The Economist* this way:

*when it comes to choosing between policies: the policy that performed best in a trial...will later be doomed to disappoint. To illustrate this the researchers turn to a trial that assesses the most effective ways of encouraging people to donate to charity, by combining requests for specific donations with promises to match the initial contribution. The researchers find that if the charity chooses the method that does best in a trial, it will always overestimate its donations.*

How would you, as a statistician, explain the winner’s curse?

10. A major U.S. bank uses 5,000 software applications. An analysis of a randomly selected month found: 90 percent of these applications have a glitch; if an application has a glitch, there is a 60% chance that a problem ticket will be generated; and if the application has no glitches, there is nonetheless a 20% chance that a problem ticket will be generated. If a problem ticket is generated, what is the probability that there really is a software glitch?

11. In a medical “pool testing” of blood samples from 25 people, the 25 samples are mixed together into a batch (or pool) and tested together. If the test comes back negative, all 25 samples are known to be negative. If the test comes back positive, then the 25 people are tested individually. If the probability of a positive test result is 0.01, what is the expected value of the number of tests if pool testing is used for a group of 25 people?

12. Identify the error(s)

*We calculated the fraction  $p$  of the soccer players born in the first six months of the year by dividing the number born from January to June by the total number of players.*

$$p = \frac{\text{players born January to June}}{\text{total number of players}}$$

*Then we set up a one-variable proportion test to figure out whether the fractions born in the first six months and last six months were equal ( $p = 0.5$ ) or not equal ( $p \neq 0.5$ ). We calculated the  $Z$  value by using*

*the equation* 
$$Z = \frac{X - \mu}{p(1-p)/\sqrt{n}}$$

13. Mahjong is played with a set of 144 randomly shuffled tiles, including 16 wind tiles, consisting of 4 East tiles, 4 South tiles, 4 West tiles, and 4 North tiles. In Taiwanese Mahjong, each of four players is dealt 16 tiles at the start of the game. If you are one of the four players, what is the probability that

- a. None of the 16 tiles you are dealt will be a wind tile
  
  
  
  
  
  
  
  
  
  
- b. None of the four players will be dealt a wind tile
  
  
  
  
  
  
  
  
  
  
- c. You will be dealt all 16 wind tiles
  
  
  
  
  
  
  
  
  
  
- d. The first 4 tiles you are dealt will be 1 East tile, 1 South tile, 1 West tile, and 1 North tile in any order

14. a. Explain why this argument either does or does not make sense to you:

*The distribution of individual test scores is bi-modal. This means that you cannot do a t-test of the sample mean of all test scores, no matter how large the sample, because that test is based on the assumption that individual scores are normally distributed.*

b. What is problematic about this cross-section regression model?

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$$

where  $Y$  = company earnings,  $X_1$  = company assets,  $X_2$  = fraction of company sales made inside the U.S.,  $X_3$  = fraction of company sales made outside the U.S.

15. Which of the following statements are true and which are false?

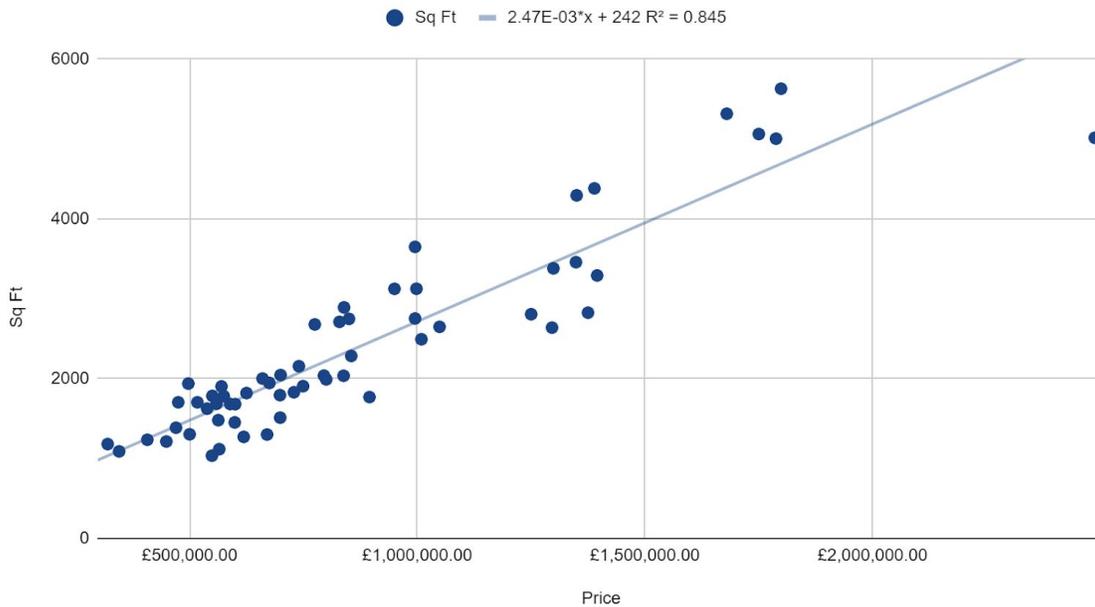
- a. The 95% confidence interval represents the probability that the null hypothesis is true.
- b. *Ceteris paribus*, if the sample size were larger, the width of the 95% confidence interval would be smaller.
- c. A 99% confidence interval would be narrower than the 95% confidence interval.
- d. 95% of the sample values are inside the 95% confidence interval.

16. You are trying to answer a multiple choice question on a standardized test. There are three choices. If you get the question right, you gain one point; if you get it wrong, you lose 1/4 point. Assume you have no idea what the right answer is, so you pick one of the three answers at random. What is the expected value of the number of points you will get on this question?

17. A regression model was constructed to predict the sale price of homes in Claremont:

*We looked at the list (asking) prices for 60 homes that are for sale in Claremont. We wrote down the square footage and price of the property and used that information to make a scatterplot with a line of least regression. The relationship between square footage and price of the listing was positive. The equation is  $y = 242 + 0.00247x$ , where  $y$  = square footage and  $x$  = list price.*

Sq Ft vs Price



How would you improve this study and the report of the results?

18. An online college statistics class asked students to apply a chi-square test to these observed and expected values, using a 5% significance level:

<i>Observed</i>	41	38	20	41	27
<i>Expected</i>	39	37	19	38	26

Why is this question flawed?

19. a. An online college statistics class asked students this question:

*A true-false quiz with 10 questions was given to 100 students in a statistics class. Following is the distribution of the scores. Find the mean score and interpret the result. Round the answers to two decimal places as needed.*

Score	5	6	7	8	9	10
Number of students	5	15	33	28	12	7

What is wrong with the given answer? (Hint: Don't check the average score; it really is 7.48)

*The mean score is 7.48. This means that if we were to give this quiz to more and more students, the average score for these students would approach 7.48.*

b. An online college statistics class asked students to use 25 observations to test the null hypothesis that the mean breaking strength of a material is  $\mu = 50$  psi. What is wrong with this interpretations of the results?

*With a one-tailed hypothesis  $\mu > 50$ , we can conclude that the mean breaking strength is greater than 50. With a two-tailed hypothesis, we will not know whether the mean breaking strength is less than or greater than 50.*

20. Why are you skeptical of this advertisement from the Motley Fool investment advisory service that appeared on Instagram on January 5, 2021, and boasted of their stock market recommendation for Zoom?

