

Final Examination (150 minutes)

No calculators, internet, or large language models allowed. Just set up your answers, for example, $P = 49/52$. You can buy extra time at a price of 1 point a minute; for example, if your test is handed in 10 minutes after the scheduled finish time, 10 points will be subtracted from the test score. **Illegible answers will be ignored.**

1. After the Cleveland Cavaliers won the National Basketball Association (NBA) championship in 2016, Dominique Wilkins, a basketball executive and Hall of Fame player, was asked, “What do you think the biggest obstacle will be for the Cavs to repeat as champs?” He answered: “Everyone is chasing them. It’s always tougher the second year because you’ve got a target on your back.” Provide a purely statistical explanation for the fact that so few teams repeat as champions.

2. Identify the most appropriate statistical test; for example, a two-sample difference-in-means test.
 - a. Daily trading volume tends to increase in December for stocks that have suffered large price declines between January and November.

 - b. GPAs tend to be higher for the players on women’s Division III college basketball teams than for the players on men’s Division III college basketball teams.

 - c. Holding earnings, risk, and growth rates constant, companies that pay higher dividends tend to have higher stock prices.

 - d. The prices of the 30 stocks in the Dow Jones Industrial Average tend to increase between the close of trading each day and the opening of trading the next day.

 - e. Major League Baseball players elected to the Hall of Fame generally live longer than other players.

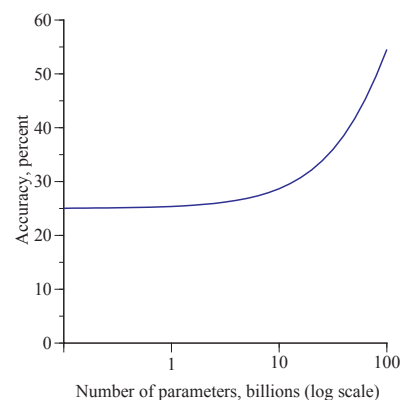
3. Identify the most appropriate statistical test; for example, a two-sample difference-in-means test.
 - a. Professional poker players tend to play looser after they lose a big pot than after they win a big pot.

 - b. Horseshoe pitchers are more likely to throw a double-ringer if they threw a double-ringer in the previous inning than if they didn’t throw a double-ringer.

 - c. A company’s stock price tends to go up on the Monday after it has an ad shown during the Super Bowl.

- d. Increases in home prices in Southern California between 2005 and 2010 tended to be higher in cities with low price/rent ratios in 2004.
 - e. Adult women are more likely to live in different ZIP codes from where they were born if their mother is a first-generation immigrant.
4. What is the most important statistical problem with each of these studies?
- a. An unusually large number of tweets of the word “happy” by Donald Trump tends to increase the level of the Dow Jones Industrial Average 3 days later.
 - b. Heart disease fatalities were reported for the 8 heart-disease categories (out of 17 total categories) in which Asian-Americans have an above-average death rate on the fourth day of the month.
 - c. A statistically significant relationship was found between household spending and wealth after income was dropped as an explanatory variable.
 - d. The benefits of studying French were demonstrated in a study that found that high school students who had taken French classes scored 80 points higher, on average, than other students on the verbal SAT.
5. A standard 52-card deck has 13 cards in each suit: 13 spades, 13 hearts, 13 diamonds, and 13 clubs. In the card game bridge, each of the four players is dealt 13 cards. If you play one game, what is the probability that you will be dealt a perfect hand—all 13 cards of one suit?

6. This figure was used to demonstrate that when the number of parameters in a large language model is increased, there is a tipping point beyond which there is an explosive improvement in the model’s performance. What unusual feature of this figure may cause the visual impression to be misleading?



7. Identify 5 distinct problems with this multiple regression model of the effect of COVID-19 on the earnings of small business owners.

$$\text{Earnings} = \alpha + \beta_1 \text{Covid} + \beta_2 \text{Race} + \varepsilon$$

Earnings = annual income from business, dollars

Covid = 1 during COVID months, 0 pre-COVID

Race = 0 if white, 1 if black, 3 if Asian

Variable	Coefficient	Standard Error	T-statistic	2-sided P-value
Constant	34,256.77	2,437.92	14.05	0.0385
Covid	1,327.38	770.95	1.72	0.0427
Race	450.22	125.43	0.28	0.000165

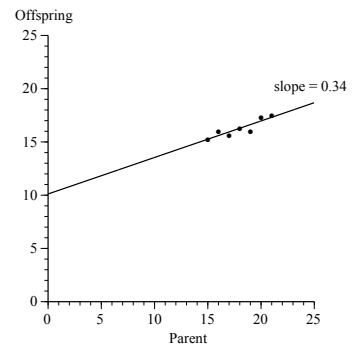
$$\varepsilon = -2.21; R^2 = 0.10$$

-
-
-
-
-

8. Here is a table of descriptive statistics for the data in the preceding study. Put these five numbers in the appropriate places: 0, 0.60, 3, 37897, 24246

Variable	Mean	Median	Standard Deviation	Minimum	Maximum
Earnings	33,830	<input type="text"/>	27,846	2,343	130,000
Covid	<input type="text"/>	0	0.50	0	1
Race	0.25	<input type="text"/>	0.72	0	<input type="text"/>
N =	<input type="text"/>				

9. Thousands of pea seeds were divided into 7 size categories and, for each category, the average size of the seeds of the offspring plants were recorded. How would you explain the fact that the fitted line does not go through the origin with a slope of one?



Pea Seed Diameters, hundredths of an inch

10. Suppose that a one-person jury has a probability p of making the correct decision and that a three-person jury decides a case by majority rule. Two of the three jurors make independent decisions, each with a probability p of making the correct decision. The third juror decides how to vote by flipping a fair coin. Which jury has the higher probability of making the correct decision?

11. On Wednesday morning, Charlie realizes that there is a final examination in 10 minutes in a class Charlie had not attended all semester. (Don't ask why.) The test consists of 10 true-false questions, each graded as follows: 1 point for a right answer, 0 points for not answering the question, and minus 2 points for a wrong answer. A score of 6 will be an *A* grade, 1-5 is a *B*, 0 is a *C*, and a negative score is an *F*. Charles does not know any of the answers and must decide how many questions to answer with random guesses.

a. What is the expected value of a guess?

b. What strategy maximizes Charlie's chances of an *A* grade? If Charlie follows this strategy, what is the probability of an *A* grade?

12. Do you agree or disagree with the following statements?

a. Since the p-value is 0.089, I accept the null hypothesis.

yes no

b. If *X*'s mean is larger than *Y*'s mean, then *X*'s median is larger than *Y*'s median.

yes no

c. In a box plot, an outlier is never inside the box.

yes no

d. Multicollinearity is when a dependent variable is correlated with the explanatory variables.

yes no

e. A model with a low R^2 has little "oomph."

yes no

13. There are four doors: one door with a \$10,000 prize hidden behind it and three doors with a bag of goldfish. You choose Door 3. No matter which door you choose, the host will then show you a bag of goldfish behind a door you did not choose (say, Door 1) and ask if you wish to switch your choice to one of the two remaining doors. What is the probability of winning the \$10,000 prize if you do switch?

14. A worldwide religious-beliefs survey found that of those people who identified with one of the five major religions, people were overall more religious in 2020 than in 2000 even though the opposite was true in each of the five individual religions. Assuming that very few people changed religions during this 20-year period, how is this possible?

15. Do you agree or disagree with the following statements about $X \sim N[\mu, \sigma]$?

- a. The median of X is equal to μ . yes no
- b. The standard deviation of $(X - \mu)/\sigma$ is 1. yes no
- c. There is about a 0.95 probability that X will be more than 2 standard deviations from μ yes no
- d. $P[X > \mu + 47] > P[X > \mu]$ yes no
- e. $P[X < \mu - 47] = P[X > \mu + 47]$ yes no

16. A researcher played the game Roshambo (rock-scissors-paper) against 120 randomly selected opponents, recording the initial move of each opponent: 33 played rock, 46 paper, and 41 scissors. What is wrong with this chi-square test of the null hypothesis that each throw is equally likely?

$$\chi_2^2 = \frac{(33-40)^2}{120} + \frac{(46-40)^2}{120} + \frac{(41-40)^2}{120}$$

17. For a woman who gives birth at age 35, the probability of having a baby suffering from Down's syndrome is $1/270$. A test of the amniotic fluid in the mother's uterus is virtually 100 percent accurate in predicting Down's syndrome, but is expensive and can cause a miscarriage. A study of the effectiveness of an inexpensive blood test that does not risk miscarriage found that in 89 percent of the Down's-syndrome cases, the test gave a positive reading, while in 75 percent of the cases without Down's syndrome the test gave a negative reading. Of those cases where there is a positive reading, what fraction are false positives?

18. A study of home prices in a suburban Rhode Island town used these variables in a multiple regression model:

Y = price

X_1 = square feet

X_2 = number of bedrooms

X_3 = number of bathrooms

X_4 = Walk Score, a rating (from 0 to 100) for a given address, based on the walking distance from that address to a variety of key amenities. A Walk Score of 70 or above is generally considered good.

What is wrong with this interpretation of the results: “The coefficient of X_4 is negative because large homes that have high prices are usually secluded from grocery stores, coffee shops, and other desirable destinations.”

19. Two researchers investigated the theory that a person’s name may influence their decisions by looking at marriages in Clay County, West Virginia, which has a very homogenous population with few people moving in or out of the county. They compared the number of married people whose first names began with the same first letter to the expected value if first letters didn’t matter:

We used the chi-square statistic $\chi^2 = \sum_{i=1}^{26} \frac{(O_i - E_i)^2}{E_i}$. To calculate O for each letter of the

alphabet, we counted the number of married couples whose first names both began with that letter and divided that number by the total number of married couples. To calculate E , we calculated the fraction of male and female names in the county that begin with that letter and then multiplied the male proportion times the female proportion. For instance, if 3% of the female first names and 3% of the male first names in a Clay County begin with the letter ‘A,’ then, by random pairings, we expect $E = 3\% \times 3\%$, or 0.09%, of the couples in the total sample to both have first names starting with ‘A.’

What error did they make in calculating the chi-square statistic?

20. What problems do you see with this figure showing how much of corporate earnings per share was paid out as dividends each year, 1972-1977? (For example, in 1972, earnings per share were \$1.53 and dividends per share were \$1.02.)

