

Final Examination (150 minutes)

No calculators allowed. Just set up your answers, for example,  $P = 49/52$ . BE SURE TO EXPLAIN YOUR REASONING. If you want extra time, you can buy time at a price of 1 point a minute; for example, if your test is handed in 10 minutes after the scheduled finish time, 10 points will be subtracted from the test score.

1. Identify the most appropriate statistical test for each of these research hypotheses. You do not need to show any formulas, just identify the test, for example, “difference-in-means t test.”
  - a. Oak trees are more are more likely to survive hurricanes than are pines trees or spruce trees.
  - b. The price of undeveloped land on the outskirts of a city depends on how close it is to the city center, and whether there are nearby water and electrical connections.
  - b. Future stock returns are higher for companies going out of the Dow Jones Industrial Average than for the companies that replace them.
  - c. Identical twins have lower lifetime incomes than do siblings who are not twins.
  - d. Each year in the Chinese lunar calendar is associated with one of five elements (fire, earth, metal, water, or wood). People born in a fire year are more likely to die of heart disease than are people born in other years.
2. Explain why you either agree or disagree with these statements regarding least squares regression:
  - a. If  $R^2 = 0$  in the model  $Y = \alpha + \beta X + \varepsilon$ , then X does not have a statistically significant effect on Y.
  - b. In the model  $Y = \alpha + \beta X + \varepsilon$ ,  $R^2 = 1$  means  $Y = X$ .
  - c. In the model  $Y = \alpha + \beta X + \varepsilon$ ,  $R^2$  is equal to the correlation coefficient squared.
  - d. In the model  $Y = AX^\beta$ ,  $\beta$  is the elasticity of Y with respect to X.



6. How would a statistician criticize this claim by a former chairman of Goldman Sachs Asset Management?  
There's an odd little thing called the five-day rule.... The rule simply states that when the main U.S. stock-market indexes show a combined positive return after the first five days of trading, the year as a whole is very likely to be a good one....

Jose Ursua, a former colleague of mine at Goldman Sachs, has run these numbers all the way back to 1928. He finds that when stocks rallied during the first five days, there was a 75.4 percent chance of a rally for the year. For the period since 1950, the probability rises to 82.9 percent. Few rules in finance are as unambiguous as that.

7. A new theory groups people into one of three personality types (X, Y, or Z) and assumes that one third of the population is in each group. A large empirical study asked people this question, "Would you rather go to the beach or the mountains for a two-week vacation?" and found that 70% of the Type X people answered "Beach," as did 50% of the Type Y people, and 30% of the Type Z people.

If Jill Jones answers "Beach," what is the probability that she is a Type X personality?



9. Data for five NBA seasons (2008-2012) were used to estimate this equation

$$Y = 2.16 + 0.62X + 10.36D$$

where  $Y$  = the team's winning percentage,  $X$  = team's average annual player salaries (dollars), and  $D = 1$  if the team has won more than half of its games in its entire existence. The two-sided p-values were 0.0158 for the coefficient of  $X$  and 0.0042 for the coefficient of  $D$ . The R-squared was 0.515. Critically evaluate: "The results suggest that teams that pay higher salaries win more games, but are not completely persuasive because franchises with all-time winning records may also have proportionately larger team salaries."

10. Sales data for single-family houses in a Dallas suburb were used to estimate the following equation:

$$P = 60,076 + 75.1S + 36.4G - 3,295A + 4,473B - 14,632T \quad R^2 = 0.84$$

(14.4)      (19.7)      (12.1)      (1,078)      (1,708)      (3,531)

where  $P$  = sale price,  $S$  = square feet of living area;  $G$  = garage square feet;  $A$  = age of house in years;  $B$  = number of baths; and  $T = 1$  if 2-story, 0 if not. The standard errors are in parentheses.

a. Which of the estimated coefficients in the regression equation are statistically significant at the 5 percent level? How can you tell?

b. Does the sign and size of each coefficient seem reasonable? In each case, explain your reasoning.

11. A study of the effect of college education on income used 1990 and 2010 U.S. Census data. The study looked at: (a) women in the 1990 Census who were between 28 and 32 years old and were also the first woman in their family to attend college; and (b) women in the 2010 Census who were between 28 and 32 years old and whose mother was the first woman in their family to attend college. The average income was \$45,000 for the women in group (a) and \$40,000 for the women in group (b), even though average income for the nation as a whole was 50% higher in 2010 than in 1990.

Is it possible that every daughter had a higher income than her mother, yet the average daughter income was lower than the average mother income? Give a specific numerical example to illustrate your argument.

12. Here is a description of an experiment designed by a psychology professor:

Let's say you were one of the participants. You walk in and sit down and he explains the rules. You are to play the role of a teacher. You will be able to punish a student for being late and reward him for being on time. Or you can choose to do nothing at all. You are presented with a series of the arrival times of a student for fifteen days in a row, one day at a time. Sometimes your student arrives late and sometimes he arrives on time.

You want to get a good result so you punish the lateness and reward the punctuality. Almost everyone in the experiment does this. What you don't know is that the student is arriving late and on time *at random*. The arrival times were all decided before the experiment started and your punishment and rewards are not influencing the student at all.

But it very much *seems* to you that your rewards and punishments are having an influence — a *bad* influence. Your student seems to do *better* after being punished, and *worse* after being rewarded. Most of the participants in this experiment concluded that punishment works better than rewards. It was the obvious conclusion, given what they experienced.

But their conclusions were wrong.

Why do you think their conclusions were wrong?

13. A researcher specified this time-series model

$$Y = \alpha + \beta X + \varepsilon$$

where Y is the annual percentage change in household spending and X is the annual percentage change in the S&P 500 index of stock prices. He reported these results

estimate of  $\alpha = 0.21$

estimate of  $\beta = 0.48$

estimate of  $\varepsilon = 1.13$

t = 2.43 for test of  $H_0: X = 0$ :

What errors do you see in these reported results?

14. When 100 MIT MBA students were offered these three subscription choices to *The Economist*,

1-year online subscription    \$59

1-year print subscription    \$125

1-year print and online subscription    \$125

16 chose the online subscription, none chose the print subscription, and 84 chose the combination print and online subscription.

When 100 other MIT MBA students were offered just two choices,

1-year online subscription    \$59

1-year print and online subscription    \$125

68 chose the online subscription and 32 chose the combination print and online subscription:

Is this observed difference in the online and print/online subscription choices between these two offers statistically persuasive?

15. A woman wrote to Dear Abby, saying that she had been pregnant for 310 days before giving birth.

Completed pregnancies are normally distributed with a mean of 266 days and a standard deviation of 16 days. What is the probability that a completed pregnancy lasts at least 310 days?

16. In a 1980 court case, a federal judge described how regression estimates of male and female salary equations could be used to determine if an employer discriminates against females.

$$\text{males: } Y = \alpha_M + \beta_M S + \gamma_M E$$

$$\text{females: } Y = \alpha_F + \beta_F S + \gamma_F E$$

where  $Y$  = annual earnings;  $S$  = number of years of schooling;  $E$  = number of years of relevant job experience. The judge also noted that in place of two separate equations, one for males and one for females, the following single equation could be estimated:

$$Y = \alpha + \beta_1 S + \beta_2 E + \beta_3 D$$

where  $Y$ ,  $S$ , and  $E$  are as defined before and  $D = 0$  if the person is a male and  $D = 1$  if the person is female.

- a. In what ways is this single equation less general than the two separate equations?

- b. What single equation could the judge have specified that is as general as the two separate equations?

17. A state lottery ticket costs \$2 and involves picking six different numbers between 1 and 46. The player wins a prize if some or all of the picks match the six randomly chosen numbers drawn by the state. What is the probability of matching *exactly* four numbers? (For example, selecting the numbers 1, 2, 3, 4, 5, 6 and the winning numbers turning out to be 3, 13, 2, 1, 6, 23.)

18. A researcher wrote that, “It remains apparent that the model needs to be manipulated and tweaked. As researchers, the next step in our study’s progress is to attempt to obtain statistically significant results.” Why is this bad advice?

19. Explaining why he was driving to a judicial conference in South Dakota, the Chief Justice of the West Virginia State Supreme Court said that, “I’ve flown a lot in my life. I’ve used my statistical miles. I don’t fly except when there is no viable alternative.” What do you suppose the phrase “used my statistical miles” means? Explain why you either agree or disagree with the judge’s reasoning.

20. The figure below is an updated version of a *New York Times* graphic that accompanied an article by David Frum titled “Welcome, Nouveaux Riches.” The figure shows a dramatic acceleration between 1980 and 1990 in the number of households earning more than \$100,000 a year. Frum wrote that, “Nothing like this immense crowd of wealthy people has been seen in the history of the planet.” What problems do you see with this graph?

