Econ 57
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Midterm Answers

1. The units on the horizontal axis are not consistent (10 years, 10 years, and then 20 years); the data are not adjusted for the total number of households; and the data are not adjusted for inflation.

2. True. It doesn't matter which ball is picked first. The probability that the second ball will be a different color is higher than the probability that the second ball will be the same color, because there are fewer balls left in the container that are the same color.

3. Here is a contingency table with a total of 27,000 women who give birth at age 35, of whom (1/270)27,000 = 100 will have babies who suffer from Down's syndrome:

	Positive Reading	Negative Reading	Total
Down	89	11	100
No Down	6,725	20,175	26,900
Total	6,814	20,186	27,000

Of the 100 Down's-syndrome babies, the blood test will give a positive reading in 0.89(100) = 89 cases and a negative reading in the remaining 11 cases. Of the 26,900 babies not suffering from Down's-syndrome, the blood test will give a negative reading in 0.75(26,900) = 20,175 cases and a positive reading in the remaining 6,725 cases. Of the 6,814 cases with positive readings, a stunning 6,725/6,814 = 0.987 are false positives.

(The false negative rate is 11/ 20,186 = 0.0005; the main benefit of this blood test is that it would screen out many women from an amniotic-fluid test that risks a miscarriage. For those who get a positive blood-test result, a follow up amniotic-fluid test would be necessary.)

4. The first die can be anything. The second die has to be one of the five remaining numbers. If it is, then the third die has to be one of the four remaining numbers. Continuing this logic, the probability is

$$1\left(\frac{5}{6}\right)\left(\frac{4}{6}\right)\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)\left(\frac{1}{6}\right)=\frac{20}{1{,}296}=0.0154$$

5. X ~N[69, 2.65] implies Z = (75 – 69)/2.65 = 2.2642 and P[Z > 2.264] = 0.01178, so that the number of males who were 6 foot 3 inches tall or taller in the early 1960s is 0.01178(30,000,000) = 353,400. Today, X ~N[70, 2.65] implies Z = (75 – 70)/2.65 = 1.8867 and P[Z > 2.264] = 0.02959, so that the number is 0.02959(30,000,000) = 887,700, a difference of 887,700 – 353,400 = 534,300.

6. Perhaps there is self-selection bias in that people who are reckless, competitive, and manipulative are attracted to this occupation.

7. The fact that they might stop before 5 games are played in irrelevant. We can simply calculate the probability that one team wins at least 3 games in a 5 game series.
   a. Using the binomial distribution:

$$P[X \geq 3] = \binom{5}{3} 0.4^3 0.6^2 + \binom{5}{4} 0.4^4 0.6^1 + \binom{5}{5} 0.4^5 0.6^0 = 0.317$$

We can't add together 3 of 5, 3 of 4, and 3 of 3 because the teams only play more than three games if neither team wins the first three games. The above calculation is correct because it doesn't matter if the teams were to keep playing after one team has won three games. To check this logic, calculate A's probability of winning the series using each of these two approaches, and see if the probability of A winning plus the probability of B winning adds to 1.0.

b. The longer the series, the less likely it is that the weaker team will win. With a 7-game series, the probability of B winning drops to 0.290.

8. The central limit theorem refers to the summation of a large number of independent draws—so the summation of the results of five dice rolls is more bell shaped than is the summation of one roll (which is a uniform distribution).

9. The probability of a particular Venus sequence is (0.39)(0.37)(0.12)(0.12). There are (4)(3)(2)(1) = 24 possible Venus sequences, since any of the four sides can appear on the first astragalus, any of the three remaining sides on the second astragalus, two on the third astragalus, and only the last side on the fourth astragalus. Thus, the probability of a Venus is 24(0.39)(0.37)(0.12)(0.12) = 0.0499, almost exactly 5%.

10. The conclusion refers to the probability that a male (or female) driver is convicted of a driving offense, but the statistics relate to the reverse probability: the probability that a driving offense involves a male motorist.