

Midterm Answers

1. The error is that when any two things increase over time, there can be a statistical correlation without any causal relationship.
2. Because every student has the same chances, each has a 1-out-of-21 probability of not being selected.
3. This is a binomial problem:

$$P[X \geq 8] = \binom{10}{8} .5^8 .5^2 + \binom{10}{9} .5^9 .5^1 + \binom{10}{10} .5^{10} .5^0 = 0.0547$$

4. This is possible. If the 90 test score raises his average test score, it will raise his overall average.

5. a.  $\frac{5}{69} \frac{4}{68} \frac{3}{67} \frac{2}{66} \frac{1}{65} \frac{1}{26} = \frac{1}{292,201,338}$

- b.  $\mu = (1 - 0.25)(\$443,000,000) \frac{1}{292,201,338} - \$2 = \$1.14 - \$2 = -0.86$

6. In general, let  $\alpha$  be your prior probability and let  $\beta$  be the probability that the persuasive juror reaches the correct conclusion. Using Bayes' Rule:

$$P[\text{guilty if juror says guilty}] = \frac{P[\text{guilty}]P[\text{juror says guilty if guilty}]}{P[\text{guilty}]P[\text{juror says guilty if guilty}] + P[\text{innocent}]P[\text{juror says guilty if innocent}]}$$

$$= \frac{\alpha\beta}{\alpha\beta + (1-\alpha)(1-\beta)}$$

In our example,

$$P[\text{guilty if juror says guilty}] = \frac{\alpha\beta}{\alpha\beta + (1-\alpha)(1-\beta)} = \frac{0.6(0.9)}{0.6(0.9) + 0.4(0.1)} = 0.931$$

$$P[\text{guilty if juror says innocent}] = \frac{\alpha(1-\beta)}{\alpha(1-\beta) + (1-\alpha)\beta} = \frac{0.6(0.1)}{0.6(0.1) + 0.4(0.9)} = 0.143$$

Interestingly, if  $\alpha = 0.5$ , this probability reduces to  $\beta$ .

Also,

$$P[\text{guilty if juror says innocent}] = \frac{P[\text{guilty}]P[\text{juror says innocent if guilty}]}{P[\text{guilty}]P[\text{juror says innocent if guilty}] + P[\text{innocent}]P[\text{juror says innocent if innocent}]}$$

$$= \frac{\alpha(1-\beta)}{\alpha(1-\beta) + (1-\alpha)\beta}$$

If  $\alpha = 0.5$ , this probability reduces to  $1 - \beta$ . That is, if you are 50-50, then your posterior probabilities are equal to the other juror's probabilities.

7. His second roll can be
  - a. two 6s ( $P = 1/36$ )
  - b. one 6 and one non-6 ( $P = (1/6)(5/6) + (5/6)(1/6) = 10/36$ ); or
  - c. two non-sixes ( $P = (5/6)(5/6) = 25/36$ )

For case (a) he has a Yahtzee and does not make a third roll. For case (b), he keeps the 6 and rolls the last die,

with a 1/6 chance of rolling the 6 needed for a Yahtzee. For case (c), he rolls both dice again, with a 1/36 chance of rolling the two 6s he needs. The total probability is

$$(1/36) + (10/36)(1/6) + (25/36)(1/36) = (36 + 60 + 25)/(36)(36) = 121/(36)(36) = 0.09336$$

8. The probabilities are the same since the Z value for a return  $X < 0$  is  $-1$  for both stocks.
9. Self-selection bias. ["Actually, Mr. President, That Art History Degree Is Pretty Helpful," Entrepreneur, February 19, 2014. [http://finance.yahoo.com/news/actually-mr-president-art-history-142000404.html?soc\\_src=mediacontentsharebuttons](http://finance.yahoo.com/news/actually-mr-president-art-history-142000404.html?soc_src=mediacontentsharebuttons)
10. Most people performing in new genres haven't been alive long enough to die at an elderly age. For example, hip hop began in the late 1970s. People who began doing hip hop in 1980 at age 20 and are still alive would be 57 years old in 2017. Anyone who died before 2017 would be younger than 57. People who began doing hip hop after 1980, at age 20 and died before 2017, would be even younger. [http://callingbullshit.org/case\\_studies/case\\_study\\_musician\\_mortality.html](http://callingbullshit.org/case_studies/case_study_musician_mortality.html)