Econ 57
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Midterm Answers

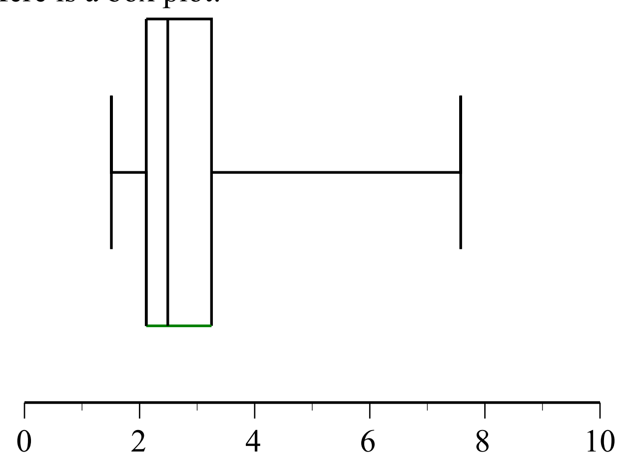
1. The first student's answer can be anything. If their excuse was untrue, the probability that the second student would select the same tire is 1/4.

2. Although 49.0% of the entire population may be male, this not necessarily mean that 49.0% of the white population is male. We need to assume that being white and being male are independent.

3. Perfectly evenly matched teams might draw every game (so that the expected value is 1 point) or they might win half their games and lose half their games, with no draws (so that the expected value is 1.5).

4. The teams with the best records may have (and in fact did) win more of their away games than did teams with worse records. He didn't really measure a home-field advantage because he didn't compare how teams did at home with how they did on the road.

5. Here are the relevant statistics: minimum = 1.510, first quartile = 2.115, median = 2.490, third quartile = 3.250, maximum = 7.580. Here is a box plot:



6. U is the standardized Z-value. U = –0.8 means that this manager is 0.8 standard deviations below the mean.

7. The probability of a match in any specified year is 1/1000. The probability of a match in four consecutive years is 1/1000^4 = 1/1,000,000,000,000, or one in a trillion.

8. The probability of 6 successes in 9 tries is given by the binomial distribution:

$$\pi = 0.50 : P[6 \text{ successes}] = \binom{9}{6} 0.50^6 0.50^3 = 0.1641$$

$$\pi = 0.67 : P[6 \text{ successes}] = \binom{9}{6} 0.67^6 0.33^3 = 0.2731$$

Now, using Bayes' Rule

$$P[\pi = 0.67 \text{ if } 6 \text{ of } 9] = \frac{P[\pi = 0.67]P[6 \text{ of } 9 \text{ if } \pi = 0.67]}{P[\pi = 0.67]P[6 \text{ of } 9 \text{ if } \pi = 0.67] + P[\pi = 0.50]P[6 \text{ of } 9 \text{ if } \pi = 0.50]}$$

$$= \frac{0.10(0.2731)}{0.10(0.2731) + 0.90(0.1641)}$$

$$= 0.1561$$

Using a contingency table,

	6 of 9	not 6 of 9	Total
Guesser	0.1641(900)	(1 - 0.1641)(900)	900
Expert	0.2731(100)	(1 - 0.2731)100	100
Total	0.1641(900) + 0.2731(100)	(1 - 0.1641)(900) + (1 - 0.2731)100	1,000

Therefore,

$$P[\pi = 0.67 \text{ if } 6 \text{ of } 9] = \frac{0.2731(100)}{0.2731(100) + 0.1641(900)}$$

$$= 0.1561$$

9. a. Yes, the expected wait is $1/\pi = 2$ for $\pi = 0.5$.
   b. Yes. The squaring of deviations makes the standard deviation more sensitive to outliers.
   c. No. For example, a 0.8 probability of a 1-point increase and an 0.2 probability of a 4-point decline has an expected value of zero.
   d. No. As the number of flips increases, the probability of exactly 50% heads declines.

10. a. Yes, the binomial distribution converges to a normal distribution as n increases for all values of π.
    b. Yes. There is a 0.95 probability of being within two standard deviations of the mean and, with a normal distribution, the mean is equal to the median.
    c. Yes. The interquartile range is the difference between the first and third quartiles and these are the edges of the box.
    d. Yes. These are Bernoulli trials with $n = 6$, $x = 2$, and $\pi = 1/6$.