

Midterm (75 minutes)

No calculators allowed. Just set up your answers, for example, $P = 49/52$. BE SURE TO EXPLAIN YOUR REASONING. If you want extra time, you can buy time at a price of 1 point a minute; for example, if your test is handed in 10 minutes after the scheduled finish time, 10 points will be subtracted from your test score.

1. In 2015, a widely circulated news story was titled, “Selfies have killed more people than sharks this year.” The subtitle was “Humans: still the world’s deadliest predator.” The story backed up its claim that humans are the deadliest predator with a report of 8 deaths from shark attacks in 2015 compared to 12 deaths related to taking ill-advised selfies, including people falling off cliffs, crashing their cars, being hit by trains, and shooting themselves while posing with guns. The story concluded, that “not only does the likelihood of being killed by a shark pale in comparison to the deadliness of selfies, it’s also a lot lower than the number of deaths caused by dog attacks and home renovations. In fact, pretty much everything you do today (particularly if it involves a car) is more likely to kill you than a shark.” Why is this report misleading?
2. In the card game War, a standard deck of 52 playing cards is divided evenly between two players. Each player turns over a card, and the player with the higher card wins both cards and puts them at the bottom of his or her deck. (Aces are high, then kings, queens, and so on; the suit does not matter.) The game continues until one player runs out of cards. If the two cards turned over are the same rank (for example, two 7s or two jacks), it is War. Each player then plays one card face down and one card face up, and the higher face-up card wins all 6 cards. What is the probability that the very first play of the game will be War?
3. “A 2001 report by Harvard University’s College Alcohol Study compared colleges that ban all alcohol to those that do not, finding that students at colleges with a ban were 30% less likely to be heavy episodic drinkers and more likely to abstain from alcohol.” What problem do you see interpreting this study?
4. A seemingly healthy woman has a physical checkup which involves a battery of tests of 20 risk factors (such as cholesterol) that might indicate a health problem. For each test, the result is flagged as abnormal if the reading is unusually high or low—specifically, if it outside a range that encompasses 95% of the readings for healthy women. Thus, if a woman is healthy, there is only a 5% chance that her reading on a test will be outside the normal range. Assuming that the test results are independent, what is the probability that a healthy woman who takes 20 such tests will have two or more abnormal readings?

5. Assume that, taking point spreads into account, a person who bets on a football game has a 50% chance of winning the bet, and that the outcome of any game does not depend on the outcomes of other games. Most betting cards work as follows: the bettor picks the winners of n games and is paid a specified amount if he or she wins all n games. For example, for a bet of x dollars, you can purchase any one of the following three cards and be paid the indicated amounts:
- $6x$ for correctly picking 3 out of 3 games.
 - $11x$ for correctly picking 4 out of 4 games.
 - $16x$ for correctly picking 5 out of 5 games.

What is the expected value of the net return from each of these bets? What does this structure of expected values suggest about bettor preferences?

6. An American roulette wheel has 38 pockets, an equal number of red and black pockets numbered 1 to 36, and two green pockets (0 and 00). The wheel is spun one direction and the ball is spun the opposite direction until the ball loses momentum and drops into one of the pockets—which is then the winning number. Assume that the game is fair with every pocket equally likely and every outcome independent.



- What is the probability that the numbers 1, 2, and 3 will be the winning numbers, in that order, on the next three spins?
- What is the probability that 23 will be the winning number at least once in the next 10 spins?
- What is the probability that a green number (either 0 or 00) will be the winning number on all five of the next 5 spins?
- What is the expected wait until a green number is the winning number?

