

Final Examination (150 minutes)

No calculators allowed. Just set up your answers, for example,  $P = 49/52$ . BE SURE TO EXPLAIN YOUR REASONING. If you want extra time, you can buy time at a price of 1 point a minute; for example, if your test is handed in 10 minutes after the scheduled finish time, 10 points will be subtracted from the test score.

1. Paul the Octopus went 8 for 8 in predicting the winner of soccer games at the 2010 World Cup. Assuming independence, use these data to test the null hypothesis that Paul has a 0.5 probability of picking the winner of a soccer game. Why might data grubbing be a problem here?
  
2. Explain why you either agree or disagree with each of these statements:
  - a. “To investigate the relationship between interest rates and stock prices, I tested the null hypothesis that there is an effect against the alternative hypothesis that there is no effect.”
  
  - b. “I rejected the null hypothesis at the 5 percent level because the p-value is larger than 0.95.”
  
  - c. “I used a t-test instead of a Z-test because the sample size was so large.”
  
  - d. “A t-value can never be negative.”
  
3. Explain why you either agree or disagree with each of these statements:
  - a. “If I flip a fair coin until I obtain a heads, on average, it will take me two flips.”
  
  - b. “I used a normal approximation to the binomial distribution because the central limit theorem tells us that for large sample sizes, the distribution of the success probability  $\pi$  is approximately normal.”
  
  - c. “If the expected value of the change in income is zero, then the change is equally likely to be positive or negative.”
  
  - d. “The probability of 5 heads when a coin is flipped 10 times is larger than the probability of 2 heads when a coin is flipped 4 times.”

4. Explain why you either agree or disagree with each of these statements:
- “An ANOVA F-test and a difference-in-means t-test give the same p-value if there are two samples.”
  - “Because the p-value is 0.01, I rejected the null hypothesis that the sample mean is -2.”
  - “Because the p-value is larger than 0.05, we can reject the alternative hypothesis at the 5 percent level.”
  - “The F-value will be negative if all the sample means are negative.”
5. You are shown three cards: one black on both sides, one white on both sides, and one white on one side and black on the other. The three cards are dropped into an empty bag and you slide one out; it happens to be black on the side that is showing. The operator of the game says, “We know that this is not the double-white card. We also know that it could be either black or white on the other side. I will bet \$5 against your \$4 that it is, in fact, black on the other side. Can the operator profit from such bets without cheating?”
6. Suppose that the percentage of the popular vote for U. S. President by the incumbent party’s candidate is related negatively to both the unemployment rate and the rate of inflation. A student learned in a macroeconomics class that there is a negative correlation between unemployment and inflation and consequently omits the rate of inflation in order to avoid a multicollinearity problem:
- $$V = \alpha + \beta U + \varepsilon$$
- Predict the effect of this omission on the estimated coefficient of the unemployment rate. Explain your reasoning so that a novice will understand your argument.
7. Explain why you either agree or disagree with these statements:
- “Two variables with a +1.00 correlation move up and down at the same time, with the same magnitude.”
  - “If the estimated slope in a simple regression model is larger than the true slope, then the estimated intercept is smaller than the true intercept.”

8. Eighteen college-educated adults between the ages of 20 and 40 were asked to rank the taste of samples of duck liver mousse, Spam, and Newman's Own canned dog food, with 1 best, 2 second best and, 3 third best.
- |          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|----------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Duck     | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 1 | 1 | 1 | 3 | 1 | 1 |
| Spam     | 3 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 2 | 3 | 2 | 2 | 2 | 2 |
| Dog food | 2 | 3 | 1 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 3 | 1 | 3 | 3 |
- Use these data to test the null hypothesis that the ratings are random.

9. The subjects were also asked to identify the sample that was dog food among these five samples: duck liver mousse, Spam, Newman's Own canned dog food, pork liver paté, and liverwurst. Three correctly identified the dog food. Calculate the exact two-sided p-value for a test of the null hypothesis that each subject has a one-fifth probability of correctly identifying the dog food.

10. A multiple regression model was used to estimate the relationship between miles per gallon  $M$ , engine size  $S$  (liters), and weight  $W$  (pounds) for 436 automobiles:

$$M = 38.6765 - 2.1414S - 0.00220W, \quad R^2 = 0.65$$

(0.1610)            (0.00026)

The standard errors are in parentheses.  $S$  and  $W$  are positively correlated.

- Does the value -0.0022 seem reasonable?
- Does the value 0.65 seem reasonable?
- Is the variable  $S$  statistically significant at the 5 percent level?
- Explain why you either agree or disagree with this conclusion: "Part of the reason that engine size has a negative effect on miles per gallon is that cars with larger engines weigh more."

11. The population of Spain is approximately four times the population of Portugal. Suppose that a random sample is taken of 1,500 Spaniards and 1,500 Portuguese. Explain why you believe that the margin for error in these polls is (a) four times larger for the Spanish poll; (b) four times larger for the Portuguese poll; (c) twice as large for the Spanish poll; (d) twice as large for the Portuguese poll; or (e) the same for both polls.

12. Survey data were collected from 120 college students on the average number of hours they slept each day during the previous semester and their GPA that semester. Sleeping hours were divided into intervals of 5-6, 6-7, 7-8, and 8-9 with respective average GPAs of 3.85, 3.43, 3.53, and 3.67. An ANOVA test gave a p-value of 0.0461. The author concluded, “The null hypothesis is rejected at the 5 percent level, which indicates we are 95% sure that sleeping hours affects GPA.” Explain the error in this reasoning.

13. A football study obtained data on the number of *sprains* caused by contact and *strains* caused by inadequate warming up during game-simulated practices and actual games:

	Sprain	Strain
Practice	9	15
Game	26	4

Set up a test of the null hypothesis that sprains and strains are unrelated to practices and games.

14. Idler and Kasl compared the number of deaths 30 days before and 30 days after various holidays. For example, they report that among the 5 “more-observant” white Protestants who died within 30 days of Christmas, 2 died before the holiday and 3 afterward. For testing the null hypothesis that deaths before and after Christmas are equally likely, they reported a p-value of 0.1875:

$$P = \binom{5}{0} 0.5^0 0.5^5 + \binom{5}{1} 0.5^1 0.5^4 = 0.1875$$

Why is this p value wrong? What is the correct p-value?

15. A researcher estimated the following multiple regression model

$$Y = \alpha + \beta_1 D + \beta_2 X + \beta_3 DX + \varepsilon$$

where Y is the percentage of the vote received by a candidate for governor, X is the state unemployment rate, and D = +1 if the candidate is the current governor and D = -1 otherwise. Interpret the coefficient of

a. X

b. DX (D times X)

16. Suppose that 1 percent of the null hypotheses tested by a researcher are false and a 99 percent are true.

Assume that the outcomes of the tests are independent and that, for each test there is a 0.05 probability of rejecting the null hypothesis if it is true and a 0.05 probability of not rejecting a null hypothesis if it is false.

a. What fraction of the null hypotheses tested by this researcher will be rejected?

b. What fraction of all the null hypotheses that this researcher rejects are in fact true?

17. A study of 30 games of Settlers of Catan compared the average resources collected by the winners of each game with the averages resources collected by all players:

average resources for all players: 49.2167; standard deviation: 13.7133

average resources for winners: 56.7667; standard deviation: 9.3281

$$t = \frac{56.7667 - 49.2167}{\sqrt{\frac{9.3281^2}{30} + \frac{13.7133^2}{30}}} = 2.0452$$

What conceptual error was made in this statistical test?

18. Why don't we deflate a country's population by its price level in order to obtain the "real population"?

19. Two professors looked at the birth months of 76 major league baseball players who committed suicide. They calculated the adjusted number of suicides in each birth month by dividing the number of suicides in that month by the total number of players with that birth month and multiplying by 1000. For example, there were a total of 638 players with January birth months; so, the adjusted January number is  $1000(6/638) = 9.4$ , rounded off to 9. The expected value in each month is  $126/12 = 10.5$  and their calculated chi-square value is

$$\chi^2 = \frac{(9-10.5)^2}{10.5} + \frac{(13-10.5)^2}{10.5} + \dots + \frac{(10-10.5)^2}{10.5} = 43.1$$

	Actual	Adjusted	Expected
January	6	9	10.5
February	7	13	10.5
March	5	8	10.5
April	5	10	10.5
May	5	9	10.5
June	6	11	10.5
July	2	3	10.5
August	19	29	10.5
September	5	8	10.5
October	7	11	10.5
November	3	5	10.5
December	6	10	10.5
Total	76	126	126

a. What is most serious error with their statistical procedure?

b. Do you think that their mistake increased or decreased the chi-square value?

c. How would you calculate the correct chi-square value?

20. A researcher compared the Internet prices of 20 Abercrombie & Fitch (A&F) male clothing items with the prices of virtually identical clothes sold by Hollister, which is owned by A&F. The average price of the 20 A&F items was \$50.13, while the average price of the 20 Hollister items was \$31.33. The t-value was 3.38 using a two-sample test and 7.01 using a matched-pair test.

a. Which test had a lower two-sided p value?

b. Explain to someone who has not taken a statistics course why the t-values could be so different.

c. The matched pair data were used to calculate a 95% confidence interval for the price difference:  $\$18.80 \pm 5.62$ . Explain why you either agree or disagree with his interpretation: "That is to say, 95% of the calculated price difference values will be in the range \$13.18 to \$24.42."