Econ 57
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Final Examination Answers

1. The natural null hypothesis is that each of these three brands is equally likely to be picked by a randomly selected college student. If so, the expected values are 72/3 = 24 for each brand. The chi-square value is 36.03:

$$\chi^2 = \frac{(13-24)^2}{24} + \frac{(12-24)^2}{24} + \frac{(47-24)^2}{24} = 33.0833$$

With 3 – 1 = 2 degrees of freedom, the probability of such a large (or larger) chi-square value is less than 0.0000000001. The null hypothesis is strongly rejected.

2. There are a total of 996 + 4,298 = 5,294 daughters, of which 996/5,294 = 0.1881 lived in different ZIP codes. Using 0.8119 in our estimate of the standard deviation, a 95% confidence interval is

$$\frac{x}{n} \pm 1.96\sqrt{\frac{p(1-p)}{n}} = 0.1881 \pm 1.96\sqrt{\frac{0.1881(1-0.1881)}{5,296}} = 0.1881 \pm 0.0105$$

3. Under the null hypothesis that the probability is 0.5, the binomial distribution gives the exact p value:

$$p = \sum_{x=2,579}^{4,298} \binom{4,298}{x} 0.5^x (1-0.5)^{n-x} = 2.98x10^{-40}$$

We double this to get the two sided p value: $6.0x10^{-40}$.

4. The Z value (without a continuity correction) is

$$Z = \frac{\frac{x}{n} - 0.5}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{\frac{2,579}{4,298} - 0.5}{\sqrt{\frac{0.5(1-0.5)}{4,298}}} = 13.1027$$

The 2-sided p-value is $3.2x10^{-39}$

5. Using a difference-in-means test with a common estimate of the success probability, p = (2,579 + 3,181)/ (4,298 + 5,935) = 0.5629

$$Z = \frac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{\frac{p(1-p)}{n_1} + \frac{p(1-p)}{n_2}}}$$

$$= \frac{\frac{2,579}{4,298} - \frac{3,181}{5,935}}{\sqrt{\frac{0.5629(1-0.5629)}{4,298} + \frac{0.5629(1-0.5629)}{5,935}}}$$

$$= 6.4493$$

The two-sided p value is 1.0×10^{-10}. This exercise can also be answered with a chi-square test with 1 degree of freedom:

	Higher Income	Lower Income
Foreign-born	2,579	1,719
California-born	3,181	2,754

The chi-square statistic is 41.5935, with is equal to the square of the Z-value for a difference-in-means test. The p values are the same.

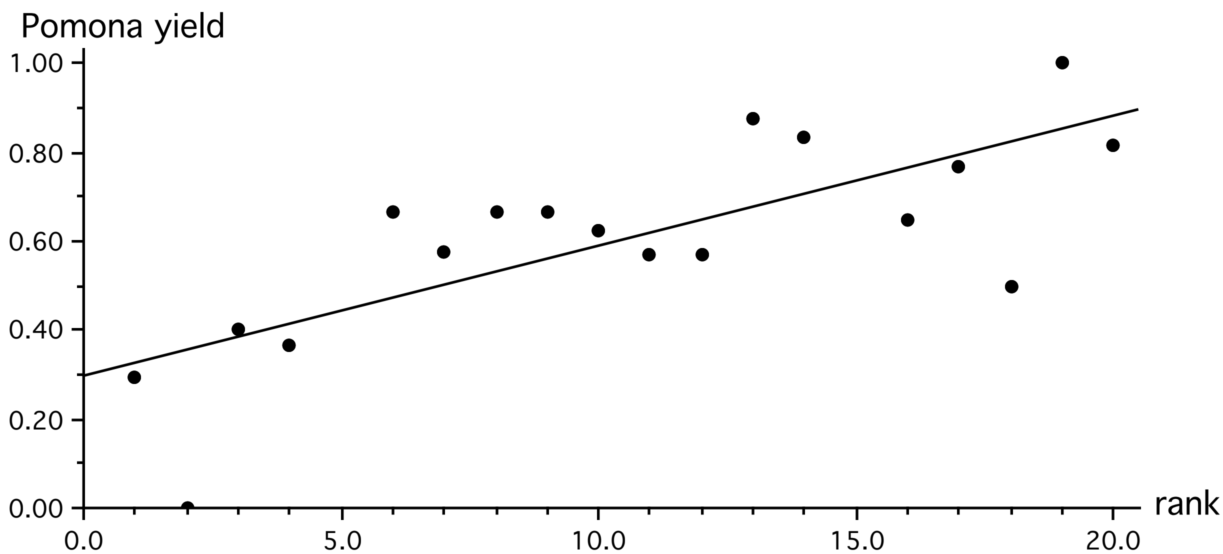
6. a. The null hypothesis is that hiring and promotions are independent of race.
   b. There might not be enough observations in the disaggregated categories.
   c. The statistical significance might be one way in some categories and the other way in the other categories, so that the differences cancel out when the data are aggregated.

7. This is a multiplication rule problem. There are initially four non-aces among the six cards and a 4/6 probability of selecting a non-ace on the first card. If a non-ace is selected, there are three non-aces among the remaining five cards and a 3/5 probability of selecting one. Thus,

   P[two non-aces] = P[first not]P[second not|first isn't] = (4/6)(3/5) = 0.4.

   Notice that the player has less than a 50 percent chance, though some might think the reverse.

8. a. Here is the regression line (notice that the Y-intercept is 0.2935):



   b. The slope and intercept were estimated by fitting the line that minimizes the sum of squared (vertical) deviations of the points from the line.
   c. There is not a perfect fit because the choice of college is affected by other factors (such as geography and special interests) in addition to the *U.S. News & World Report* rank.

9. a. Yes, because the 0.0293 coefficient implies that if the non-Pomona school's ranking increases by 1, say from 10 to 11, the fraction of these students who choose to attend Pomona will increase by 0.0293 (from $Y = 0.2935 + 0.0293(10) = 0.5865$ to $Y = 0.2935 + 0.0293(11) = 0.6158$ in our example), which is plausible.

b. Yes, because the t-value is 0.0293/0.0065 = 4.51
c. The null hypothesis is that the coefficient of X is 0; that is, that X has no effect on Y.
d. The predicted value of Y is 0.2935 + 0.0293(30) = 1.1725
e. This is incautious extrapolation, which is clearly inappropriate here since the value of Y cannot possibly be larger than 1.

10. The SOI is the z value multiplied by 10. A SOI reading of -22.8 corresponds to a z value of -2.28, which has a probability of 0.0113.

11. a. Supporters of lower speed limits would expect β to be positive (perhaps with an R-squared that is close to one). There is no persuasive reason why β should be close to one.
    b. The data show Y declining as X increases. The ordinary least squares estimate of β is negative.
    c. The peculiar choice of years 1980, 1984, and 1985 is very suspicious.

12. The people surveyed are not a randomly selected group who are asked to stay at Red Lion and then tell whether they would come back. Instead, the people surveyed are those who voluntarily chose Red Lion; this survey does not include anyone who tried Red Lion and never came back. Suppose that 1,000 people try Red Lion and that 49 decide to return and 951 do not. A survey is now made of 50 people—the 49 returnees plus one person trying Red lion for the first time. In this hypothetical example, 95.1% of the people who try Red Lion for the first time never go again, but 49 of 50 people staying at Red Lion will return.

13. It doesn't matter. Imagine that each person drew a slip of paper without looking at it. Would it matter who looked first?

14. In a classic paper, Kahnemann and Tversky (1973) cite this problem that they gave graduate students in psychology, which described the actual experience of one of the authors in advising the Israeli air force. The flight instructors were unaware of the regression argument and none of the graduate students suggested regression as a possible explanation for the observed data: "The respondents had undoubtedly been exposed to a thorough treatment of statistical regression. Nevertheless, they failed to recognize an instance of regression when it was not couched in the familiar terms of the heights of fathers and sons."

15. a. Yes, if the p-value is less than 0.01, then it is also less then 0.05.
    b. No, the ANOVA F statistic tests the null hypothesis that the population means are equal.
    c. No, if the chi-square value is 0, then the p value is 1, since this provides no evidence whatsoever against the null hypothesis.
    d. No, you must decide whether to use a one-sided or two-sided p value before you look at your data.
    e. Yes.

16. The coefficient of the dummy variable measure the extent to which extracurricular activity affects grades, holding study time constant (since study time is another explanatory variable in the equation). The coefficient of the dummy variable compares students who study equally, but one participates in an extracurricular activity in addition, and the other doesn't. (There might be an effect, for example, if the student is physically or emotionally drained by the extracurricular activity.)

17. Using Bayes' Rule,

$$P\left[\text{careless} \mid \text{fire}\right] = \frac{P[\text{careless}]P[\text{fire} \mid \text{careless}]}{P[\text{careless}]P[\text{fire} \mid \text{careless}] + P[\text{not careless}]P[\text{fire} \mid \text{not careless}]}$$

$$= \frac{0.01(0.010)}{0.01(0.010) + 0.99(0.001)}$$

$$= \frac{10}{109}$$

Using a contingency table with 100,000 homes:

	fire	no fire	total
careless	10	990	1,000
careful	99	98,901	99,000
total	109	99,891	100,000

The probability that a home destroyed by fire was occupied by a careless person is 10/109: 0.092. This 9% figure is much larger than the 1% of the total population that is careless, but it is still far from certain that a house destroyed by fire was occupied by a careless household.

18. The margin of error varies inversely with the square root of the sample size. To cut the margin of error in half, we need to quadruple the sample size. Here, 500 responses gives a margin of error of about 5% (4.47%) and 2,500 responses gives a margin of error of 2%.

19. Common sense says that these were not random samples. Those who ordered pitchers were surely planning to drink a lot and fulfilled their objectives. Pitchers do exert some psychological pressure to finish what has been paid for, but surely big drinkers would still drink a lot even if they were forced to do so by the glass or bottle.

20. This statement relies on the fallacious Law of Averages, the belief that heads must be offset by tails and good luck must be offset by bad. If anything, 25 years without a fire suggests that this is a careful family with a lower than average chance of having a fire.