Econ 58
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Midterm (75 minutes)

1. A firm's production is characterized by this Cobb-Douglas production function

$$Q = 12K^{0.4}L^{0.6}$$

Circle the correct answer to each of these 4 questions:

a. If capital K increases by 5%, what is the approximate percentage change in output Q?

0.4   0.6   1   2   4.8

b. If capital K increases by 3% and labor L increases by 4%, what is the approximate percentage change in the capital-labor ratio K/L?

-7   -1   0   1   7

c. If price P increases by 5% and output Q falls by 4%, what is the approximate percentage change in revenue PQ?

-9   -1   0   1   9

d. If capital K and Labor L both increase by 5%, what is the approximate percentage increase in output Q?

0%   2%   5%   8%   12%

2. In the constant dividend-growth model, the value of a stock that currently pays a dividend D, that will grow at an annual rate g, and whose dividends are discounted by a required rate of return R, is

$$V = \frac{D}{R \,\square\, g}$$

a. Determine the elasticity of value with respect to R: $\square = \left| \frac{\partial V}{\partial R} \frac{R}{V} \right|$

b. Determine whether the value of this elasticity □ is higher or lower for growth stocks; i.e., whether this elasticity □ is increased or decreased by an increase in g.

3. Suppose that a person's utility U depends on two items, X and Y, in this way

$$U = 3\ln[X] + \ln[Y]$$

where "ln" denotes the natural logarithm. (Be sure to show your work in answering these questions.)

a. Holding Y constant, is $\frac{\partial U}{\partial X}$, the first derivative of utility with respect to X, positive, zero, or negative?

b. Holding Y constant, is the second derivative, $\frac{\partial^2 U}{\partial X^2}$, positive, zero, or negative?

c. An indifference curve with X on the horizontal axis and Y on the vertical axis shows those combinations of X and Y that give a constant level of utility. Set utility equal to a constant amount U_0 and find the slope of the indifference curve, $\frac{dY}{dX}$, at the point X = Y = 10.

d. Interpret the value of $\frac{dY}{dX}$ determined in part c; for example, if $\frac{dY}{dX} = 5$, does this mean that Y is 5 times X, that the utility from Y is 5 times the utility from X, or what?

4. Consider this cost function that shows how a firm's cost C depends on its output Q, and this inverse demand function that shows how the market price P is related to the quantity Q:

$$C = 100Q^{1.5}$$

$$P = 7500Q^{-0.5}$$

a. What value of Q maximizes profit π = revenue - cost = PQ - C?

b. What is the value of marginal cost, $MC = \dfrac{dC}{dQ}$ at the profit-maximizing output?

c. What is the value of marginal revenue, $MR = \dfrac{d(\text{revenue})}{dQ}$ at the profit-maximizing output?

d. Explain the economic reason why MC and MR are related at the profit-maximizing level of output.

5. The initial cost of planting a stand of trees is C = 500 and the dollar value of the volume (in cubic meters) of marketable timber at time t is

$$V[t] = 100t^{0.9}$$

a. Using a continuously compounded interest rate R = 0.05, what is the net present value of the stand if it is harvested at time t?

b. Determine the optimal harvest date t that maximizes the net present value.

c. Show that your answer to part b is a maximum, and not a minimum.

d. Explain the economic meaning of your answer to part b.