Econ 58
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Midterm (75 minutes)

1. Consider a monopolist facing this demand curve Q = a - bP, and with production costs C = e + fQ, where a, b, e, and f are positive parameters.
   a. Determine the price P that maximizes profit Π = PQ - C.

   b. Show that this is a maximum, not a minimum.

   c. If b were higher, would the profit-maximizing price be higher, lower, or unchanged?

   d. Explain why your answer to Part (c) is reasonable, without using any math.

2. A researcher wants to estimate the percentage change in the overall cigarette tax rate from these data:
   %R = percentage change in tax revenue = 3%, where tax revenue is equal to the tax rate multiplied by the dollar value of cigarettes sold
   %P = percentage change in the price of cigarettes = 3%
   %Q = percentage change in the number of cigarettes sold = 3%

   Write down a model that will allow you to estimate the percentage change in the tax rate.

3. Suppose that a firm produces an identical product in two different locations with these cost functions

$$C_1 = 10 + 8Q_1 + 1Q_1^2$$

$$C_2 = 10 + 2Q_2 + 2Q_2^2$$

where C_1 and C_2 are the respective costs of producing quantities Q_1 and Q_2.

a. Use the Lagrangian method to find the quantities Q_1 and Q_2 that minimize the total cost $C = C_1 + C_2$ of producing total output $Q_1 + Q_2 = 12$.

b. What is the marginal cost of production in the first location, $\dfrac{dC_1}{dQ_1}$?

c. What is the marginal cost of production in the second location, $\dfrac{dC_2}{dQ_2}$?

d. Explain why the marginal cost in Part (b) is larger than, smaller than, or equal to the marginal cost in Part (c), using economic logic, not math.

4. In a simple money inventory model, people hold money that earns no interest and bonds that earn an interest rate R, but incur a transaction cost C whenever the bonds are bought or sold. An amount of income Y is received at evenly spaced intervals, at which point total assets (money plus bonds) is equal to income, and spent at a constant rate until total assets equal 0 and income Y is received again. It can be shown that if n bond transactions are made during each period, the optimal plan is to buy (n - 1)Y/n worth of bonds at the beginning of the period and then sell bonds worth Y/n at intervals equal to 1/n of the length of the period. Interest revenue net of transactions costs each period works out to be

$$Z = \frac{(n - 1)RY}{2n} - nC$$

   a. Determine the formula for the value of n that maximizes Z.

   b. What is the value of n if $R = 0.05$, $Y = 1000$, and $C = 1$?

5. Consider a model in which capital cannot be rented, but must be purchased at a price P_K per unit of capital. This capital will be used with labor to produce output every year. Assume, for simplicity, that the capital lasts forever; the firm uses the same amount of labor L and capital K to produce the same amount of output $Q = F[K, L]$ every period; the wage rate w will be constant indefinitely; and that the firm can sell as much output as it wants at a price P that will also be constant indefinitely.

   a. Write down an explicit model that a firm can use to determine how much capital to purchase and how much labor to employ. You do not need to determine the optimal capital stock; you do need to write down explicit equation(s) that can be used for this purpose and explain how to use them.

   b. If the firm purchases the optimal capital using your model, how will the ratio of the marginal product of capital to the marginal product of labor be related to the wage rate and other variables in your model?