Econ 58
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Final Exam Answers

1. a. The profit-maximizing level of L is

$$\pi = PAL^{2/3} - B - WL$$

$$\frac{d\pi}{dL} = \frac{2}{3}PAL^{-1/3} - W = 0 \text{ at}$$

$$L = \left(\frac{\frac{2}{3}A}{W/P}\right)^3$$

b. To verify that this is a maximum, we check that the second derivative is negative:

$$\frac{d^2\pi}{dL^2} = -\frac{2}{9}PAL^{-4/3} < 0$$

c. This depends on the values of A, B, W and P.

d. The equation for L in part (a) shows that the profit-maximizing level of L is unrelated to B. This is because B is a fixed cost.

e. The equation for L in part (a) shows that the profit-maximizing level of L is positively related to P: for given productivity and wage rate, the higher the price at which output can be sold, the more labor is demanded.

2. f. We can rewrite the equation for the profit-maximizing level of L as a relationship between W/P and L:

$$\frac{W}{P} = \frac{2}{3}AL^{-1/3}$$

g. The right-hand side of this equation is the marginal product of labor.

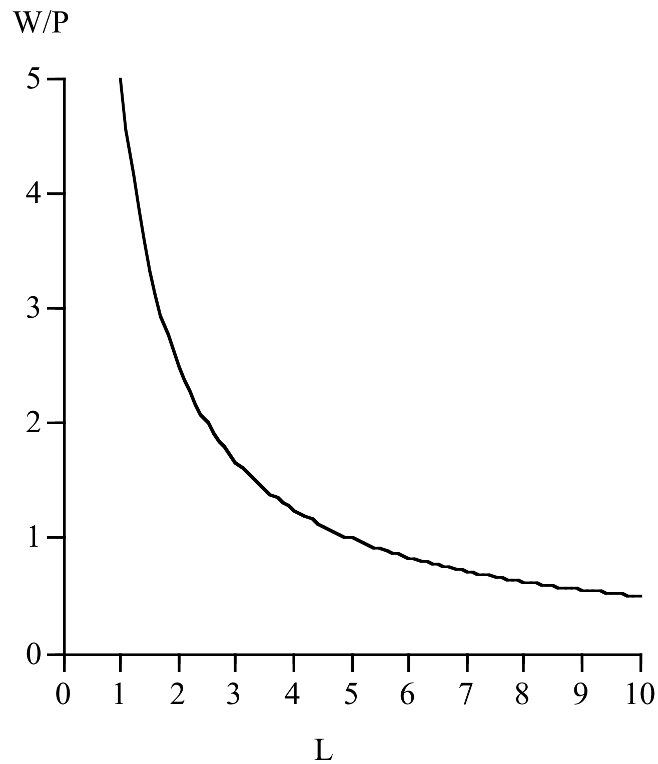
h. The slope is negative because the first derivative is negative:

$$\frac{d(W/P)}{dL} = -\frac{2}{9}AL^{-4/3} < 0$$

i. The relationship is curved upward because the second derivative is positive:

$$\frac{d^2(W/P)}{dL^2} = \frac{8}{27}AL^{-7/3} > 0$$

j. Here is a sketch for A = 2.5:



3. The present value equation is

$$\$41 = \$2.9 + \frac{\$2.9}{(1+R)^1} + \frac{\$2.9}{(1+R)^2} + \ldots + + \frac{\$2.9}{(1+R)^{24}}$$

The solution (by trial-and-error) is R = 0.0571 (5.71%)

4. a. Substituting:

$$y_1 = a_1 + b_1 y_1 + d_2 + e_2 y_2 - e_1 y_1$$
$$y_2 = a_2 + b_2 y_2 + d_1 + e_1 y_1 - e_2 y_2$$

Collecting terms

$$(1 - b_1 + e_1) y_1 = a_1 + e_2 y_2$$
$$(1 - b_2 + e_2) y_2 = a_2 + e_1 y_1$$

Solving:

$$(1 - b_1 + e_1) y_1 = a_1 + e_2 \frac{a_2 + e_1 y_1}{1 - b_2 + e_2}$$
$$(1 - b_2 + e_2)(1 - b_1 + e_1) y_1 = (1 - b_2 + e_2) a_1 + e_2 (a_2 + e_1 y_1)$$
$$\left((1 - b_2 + e_2)(1 - b_1 + e_1) - e_2 e_1\right) y_1 = (1 - b_2) a_1 + e_2 (a_1 + a_2)$$
$$y_1 = \frac{(1 - b_2) a_1 + e_2 (a_1 + a_2)}{(1 - b_2 + e_2)(1 - b_1 + e_1) - e_2 e_1}$$

Some comparative-statics multipliers are

$$\frac{\partial y_1}{\partial a_1} = \frac{1 - b_2 + e_2}{(1 - b_2 + e_2)(1 - b_1 + e_1) - e_2 e_1}$$
$$\frac{\partial y_1}{\partial a_2} = \frac{e_2}{(1 - b_2 + e_2)(1 - b_1 + e_1) - e_2 e_1} > 0, \text{ but } < \frac{\partial y_1}{\partial a_1}$$

Notice that the comparative-static multiplier $M = \frac{\partial y_1}{\partial a_1}$ is positively related to e_2, negatively related to e_1

$$\frac{\partial M}{\partial e_1} = \frac{-\left((1-b_2+e_2)-e_2\right)}{\left((1-b_2+e_2)(1-b_1+e_1)-e_2e_1\right)^2}$$

$$= \frac{-(1-b_2)}{\left((1-b_2+e_2)(1-b_1+e_1)-e_2e_1\right)^2} < 0$$

$$\frac{\partial M}{\partial e_2} = \frac{\left((1-b_2+e_2)(1-b_1+e_1)-e_2e_1\right)-(1-b_2+e_2)\left((1-b_1+e_1)-e_1\right)}{\left((1-b_2+e_2)(1-b_1+e_1)-e_2e_1\right)^2}$$

$$= \frac{(1-b_1)(1-b_2)+(1-b_2)e_1+(1-b_1)e_2-(1-b_2+e_2)(1-b_1)}{\left((1-b_2+e_2)(1-b_1+e_1)-e_2e_1\right)^2}$$

$$= \frac{(1-b_2)e_1}{\left((1-b_2+e_2)(1-b_1+e_1)-e_2e_1\right)^2} > 0$$

b. Imports are a leakage that is not fully offset by induced exports; thus the multiplier would be larger if both e_1 and e_2 were equal to 0.

c. To confirm Part (b), let N be the multiplier if $e_1 = e_2 = 0$:

$$M = \frac{1-b_2+e_2}{(1-b_2+e_2)(1-b_1+e_1)-e_2e_1}$$

$$N = \frac{1-b_2}{(1-b_2)(1-b_1)}$$

$$N > M$$

$$\frac{1-b_2}{(1-b_2)(1-b_1)} > \frac{1-b_2+e_2}{(1-b_2+e_2)(1-b_1+e_1)-e_2e_1}$$

$$(1-b_2)\left((1-b_2+e_2)(1-b_1+e_1)-e_2e_1\right) > (1-b_2+e_2)(1-b_2)(1-b_1)$$

$$(1-b_2)(1-b_2+e_2)e_1 - (1-b_2)e_2e_1 > 0$$

$$(1-b_2)^2 > 0$$

5. Setting demand equal to supply, we have

$$d - eP_t = -a + bP_t + cP_{t-1}$$

$$(e+b)P_t = a + d - cP_{t-1}$$

The dynamic equilibrium price P* is at

$$(e+b)P^* = a + d - cP^*$$

$$P^* = \frac{a+d}{e+b+c}$$

For dynamic stability, we use these two equations:

$$(e+b)P_t = a + d - cP_{t-1}$$

$$(e+b)P^* = a + d - cP^*$$

Taking the difference:

$$(e + b)(P_t - P^*) = -c(P_{t-1} - P^*)$$
$$(P_t - P^*) = \left(\frac{-c}{e + b}\right)(P_{t-1} - P^*)$$

which is cyclical and dynamically stable if and only if $c/(e + b) < 1$.

6. A contingency table will work. Using Bayes' Rule:

$$\begin{aligned} P[HH \text{ if } 5] &= \frac{P[HH]P[5 \text{ if } HH]}{P[HH]P[5 \text{ if } HH] + P[notHH]P[5 \text{ if } notHH]} \\ &= \frac{(1/20)(1)}{(1/20)(1) + (19/20)(1/32)} \\ &= 0.627 \end{aligned}$$

7.

a. $\dfrac{dB}{de} = \dfrac{dX}{de} - e\dfrac{dM}{de} - M$

b. $\dfrac{dB}{de} = \left(\dfrac{dX}{de}\dfrac{e}{X}\right)\left(\dfrac{X}{e}\right) - \left(\dfrac{dM}{de}\dfrac{e}{M}\right)M - M$ Setting X = eM. we have $\dfrac{dB}{de} = \left(\dfrac{dX}{de}\dfrac{e}{X}\right)M - \left(\dfrac{dM}{de}\dfrac{e}{M}\right)M - M$. Now dB/de > 0 if:

$$\left(\frac{dX}{de}\frac{e}{X}\right)M - \left(\frac{dM}{de}\frac{e}{M}\right)M - M > 0$$
$$\left(\frac{dX}{de}\frac{e}{X}\right) - \left(\frac{dM}{de}\frac{e}{M}\right) - 1 > 0$$
$$\left(\frac{dX}{de}\frac{e}{X}\right) - \left(\frac{dM}{de}\frac{e}{M}\right) > 1$$

This is the Marshall Lerner condition, that the sum of the absolute values of the import and export elasticities must be greater than 1 for a depreciation of the currency to increase net exports.

8.

a. Taking the partical derivative:

$$\frac{\partial V}{\partial R} = \frac{-D}{(R - g)^2}$$

Thus the elasticity is

$$\begin{aligned} \varepsilon &= \left|\frac{\partial V}{\partial R}\frac{R}{V}\right| \\ &= \left|\left(\frac{-D}{(R - g)^2}\right)\left(\frac{R}{D/(R - g)}\right)\right| \\ &= \frac{R}{R - g} \end{aligned}$$

b. The partical derivative of the elasticity with respect to g is positive

$$\frac{\partial \varepsilon}{\partial g} = \frac{R}{(R - g)^2} > 0$$

Thus the value of a growth stock is more sensitive to changes in R.

9. As with the in-class election example, the payoff is the probability of making the shot:

	shooter	
goalie	left	right
left	0.3	1.0
right	1.0	0.2

Suppose the shooter has a probability p of shooting left and that the goalie has a probability q of jumping left. The expected value of the probability is

$$\begin{aligned}\pi &= 0.3pq + 1(1-p)q + 0.2(1-p)(1-q) + 1p(1-q)\\ &= 0.3pq + q - pq + 0.2 - 0.2p - 0.2q + 0.2pq + p - pq\\ &= 0.2 + 0.8q + 0.8p - 1.5pq\end{aligned}$$

Taking the derivatives

$$\begin{aligned}\frac{\partial \pi}{\partial p} &= 0.4q - q - 0.2(1-q) + (1-q)\\ &= 0.8 - 1.5q\\ \frac{\partial \pi}{\partial q} &= 0.4p + (1-p) - 0.2(1-p) - p\\ &= 0.8 - 1.5p\end{aligned}$$

The optimal strategies (where the derivatives equal 0) are p = q = 0.8/1.5 = 0.53.

The expected value of the probability of scoring is

$$\begin{aligned}\pi &= 0.4(0.533)(0.533) + 1(0.467)(0.533) + 0.2(0.467)(0.467) + 1(0.533)(0.467)\\ &= 0.655\end{aligned}$$

10. a. We can impose the constraint $\alpha_1 + \alpha_2 = 1$ either by substituting to eliminate α_1 or α_2, or by using the Lagrangian method:

$$\sigma^2 = \alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_1^2 - 2\alpha_1\alpha_2\sigma_1^2 + \lambda(1 - \alpha_1 - \alpha_2)$$

$$\frac{\partial \sigma^2}{\partial \alpha_1} = 2\alpha_1\sigma_1^2 - 2\alpha_2\sigma_1^2 - \lambda$$

$$\frac{\partial \sigma^2}{\partial \alpha_2} = 2\alpha_2\sigma_1^2 - 2\alpha_1\sigma_1^2 - \lambda$$

Setting the partial derivatives equal to 0:

$$\begin{aligned}2\alpha_1\sigma_1^2 - 2\alpha_2\sigma_1^2 &= 2\alpha_2\sigma_1^2 - 2\alpha_1\sigma_1^2\\ \alpha_1 - \alpha_2 &= \alpha_2 - \alpha_1\\ 2\alpha_1 &= 2\alpha_2\\ \alpha_1 &= \alpha_2\end{aligned}$$

Thus $\alpha_1 = \alpha_2 = 0.5$

b. it must be a minimum since the variance cannot be less than 0.

c. The minimum value of the portfolio variance is

$$\begin{aligned}\sigma^2 &= 0.5^2\sigma_1^2 + 0.5^2\sigma_1^2 - 2(0.5^2)\sigma_1^2\\ &= 0\end{aligned}$$

Thus, when two assets are perfectly negatively correlated, a perfectly risk-free hedge is possible.