Econ 58
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Final Exam Answers

1. a. This equation works: $Y_t = Y_0 e^{0.05t}$

   b. If we let spending be C, then $C_t = 0.9Y_t = 0.9Y_0 e^{0.05t} = C_0 e^{0.05t}$ and spending grows at a 5% rate too.

   c. If we let saving be S, then $S_t = 0.1Y_t = 0.1Y_0 e^{0.05t} = S_0 e^{0.05t}$ and saving grows at a 5% rate too.

2. We can use the objective function

$$Z = U = 6(10L)^{1/4} R^{2/4} + \lambda\left(24 - L - R\right)$$

Take the partial derivatives of this utility function with respect to L and R, and set these first derivatives equal to 0:

$$\frac{\partial Z}{\partial L} = 1.5(10)(10L)^{-3/4} R^{2/4} - \lambda = 0.25\frac{U}{L} - \lambda$$

$$\frac{\partial Z}{\partial R} = 3(10L)^{1/4} R^{-2/4} - \lambda = 0.50\frac{U}{R} - \lambda$$

Setting the first derivatives equal to 0 gives R = 2L:

$$0.25\frac{U}{L} = \lambda$$

$$0.50\frac{U}{R} = \lambda$$

$$0.25\frac{U}{L} = 0.50\frac{U}{R}$$

$$\frac{R}{L} = 2$$

This condition R = 2L combined with the time constraint R + L = 24 gives L = 8 and R = 16:

$$2L + L = 24$$

$$L = 8$$

3. Substitution gives a dynamic equation

$$Y = a + bY + cY_{-1}$$

Dynamic equilibrium at Y^e requires $Y = Y_{-1} = Y^e$:

$$Y^e = a + bY^e + cY^e$$

$$= \frac{a}{1 - b - c}$$

   a. The comparative-static effect on dynamic equilibrium of a change in a is

$$\frac{\Delta Y}{\Delta a} = \frac{1}{1 - b - c}$$

We can investigate dynamic stability by subtracting the dynamic equilibrium equation $Y^e = a + bY^e + cY^e$ from the dynamic equation $Y = a + bY + cY_{-1}$:

$$Y - Y^e = b\left(Y - Y^e\right) + c\left(Y_{-1} - Y^e\right)$$

$$= \frac{c}{1 - b}\left(Y_{-1} - Y^e\right)$$

b. The model is monotonically stable if b < 1 and c < 1 - b (which implies b + c < 1); e.g., b = 0.5, c = 0.3
c. The model monotonically unstable if b < 1 and c > 1 - b (which implies b + c > 1); e.g., b = 0.7, c = 0.5.
d. The model is cyclically stable if b > 1 and c < |1-b|; for example, b = 1.5 and c = 0.3
e. The model is cyclically unstable if b > 1 and c > |1-b|; for example, b = 1.2 and c = 0.3

4. Using Bayes' Rule,

$$P\left[\pi = 0.49 \text{ if } \frac{x}{n} = \frac{505}{1000}\right] = \frac{P[\pi = 0.49]P\left[\frac{x}{n} = \frac{505}{1000} \text{ if } \pi = 0.49\right]}{P[\pi = 0.49]P\left[\frac{x}{n} = \frac{505}{1000} \text{ if } \pi = 0.49\right] + P[\pi = 0.51]P\left[\frac{x}{n} = \frac{505}{1000} \text{ if } \pi = 0.51\right]}$$

$$= \frac{0.5(0.016084)}{0.5(0.016084) + 0.5(0.023996)}$$

$$= 0.4013$$

$$P\left[\pi = 0.51 \text{ if } \frac{x}{n} = \frac{505}{1000}\right] = 1 - P\left[\pi = 0.49 \text{ if } \frac{x}{n} = \frac{505}{1000}\right]$$

$$= 0.559$$

We can also use a contingency table where we consider two possible situations, one with π = 0.49 and the other with π = 0.51, and x/n = 505/1000 is one of many results that might have been obtained:

	x/n = 505/1000	x/n = ?/1000	x/n = ?/1000	
π = 0.49	0.016084			1
π = 0.51	0.023996			1
Total	0.016084 + 0.023996			2

Therefore,

$$P\left[\pi = 0.49 \text{ if } \frac{x}{n} = \frac{505}{1000}\right] = \frac{0.016084}{0.016084 + 0.023996}$$

$$= 0.4013$$

5. Here is the payoff matrix (with the payoffs written as SportsU's probability of winning, opponent's probability of winning):

		Opponent's Defense	
		run	pass
SportsU's	run	0.3, 0.7	0.8, 0.2
play	pass	0.9, 0.1	0.2, 0.8

a. There is no noncooperative equilibrium with a fixed strategy.
b. With probabilistic strategies, SportsU's probability of winning is

$$\begin{aligned} Z &= 0.3pq + 0.8p(1-q) + 0.2(1-p)(1-q) + 0.9(1-p)q \\ &= 0.3pq + 0.8p - 0.8pq + 0.2 - 0.2p - 0.2q + 0.2pq + 0.9q - 0.9pq \\ &= 0.2 + 0.6p + 0.7q - 1.2pq \end{aligned}$$

The first derivative with respect to p is

$$0 = \frac{\partial Z}{\partial p} = 0.6 - 1.2q \Rightarrow q = 0.5$$

The opponent's probability of winning is

$$\begin{aligned} Y &= 0.7pq + 0.2p(1-q) + 0.8(1-p)(1-q) + 0.1(1-p)q \\ &= 0.7pq + 0.2p - 0.2pq + 0.8 - 0.8p - 0.8q + 0.8pq + 0.1q - 0.1pq \\ &= 0.8 - 0.6p - 0.7q + 1.2pq \end{aligned}$$

The first derivative with respect to q is

$$0 = \frac{\partial Y}{\partial q} = -0.7 + 1.2p \Rightarrow p = \frac{0.7}{1.2} = 0.583$$

6. a. By inspection, α
   b. β
   c. λP is the price the seller receives, net of the realtor commissions.
   d. The market equilibrium price is

$$B(\lambda P)^{\beta} = AP^{-\alpha}$$

$$B\lambda^{\beta}P^{\beta} = AP^{-\alpha}$$

$$P^{\alpha+\beta} = \frac{A}{B\lambda^{\beta}}$$

$$P = \left(\frac{A}{B\lambda^{\beta}}\right)^{\frac{1}{\alpha+\beta}}$$

7. a. Rewriting the market equilibrium price as

$$P = \left(\frac{A}{B}\right)^{\frac{1}{\alpha+\beta}}\left(\lambda^{\frac{-\beta}{\alpha+\beta}}\right)$$

   we see that the elasticity of price with respect to λ is $\frac{-\beta}{\alpha+\beta}$.
   b. If $\beta = 0$, the value of the elasticity of price with respect to λ is 0.
   c. If supply is perfectly inelastic, then changes in the real-estate commission have no effect on the market price. Changes in the commission affect the amount sellers receive, but not the amount buyers pay.

8. a. In matrix form:

$$\begin{bmatrix} 1-E_Y & -E_R \\ L_Y & L_R \end{bmatrix}\begin{bmatrix} Y \\ R \end{bmatrix} = \begin{bmatrix} E_0 \\ M_0 - L_0 \end{bmatrix}$$

   b. The model does not have a unique solution if the determinant is 0:

$$\begin{vmatrix} 1-E_Y & -E_R \\ L_Y & L_R \end{vmatrix} = (1-E_Y)(L_R) + (E_R)(L_Y) = 0$$

   c. If this model has no unique solution, it is because either (a) the two equations are contradictory; or (b) one of the two equations simply restates the first equation. Graphically, the two equations do not have a unique intersection because they are either (a) parallel and never cross; or (b) lie on top of each other.

9. The present value is

$$P = -50 + 100t_1^{0.9}e^{-Rt_1} - 50e^{-Rt_1} + 100t_2^{0.9}e^{-R(t_1+t_2)}$$

   Take the derivatives with respect to the two harvest times:

$$\frac{\partial P}{\partial t_1} = 90t_1^{-0.1}e^{-Rt_1} - 100Rt_1^{0.9}e^{-Rt_1} + 50Re^{-Rt_1} - 100Rt_2^{0.9}e^{-R(t_1+t_2)}$$

$$\frac{\partial P}{\partial t_2} = 90t_2^{-0.1}e^{-R(t_1+t_2)} - 100Rt_2^{0.9}e^{-R(t_1+t_2)}$$

Setting the first derivatives equal to 0, we have two equations that can be solved for the values of the two harvest times

$$90t_1^{-0.1}e^{-Rt_1} = 50Rt_1^{0.9}e^{-Rt_1} + 100Rt_2^{0.9}e^{-R(t_1+t_2)}$$

$$t_2 = \frac{R}{0.9}$$

10. 1. Target GDP grows at a constant rate: $Y_t^* = Y_0^*(1+g)^t$, where g is the positive growth rate.
2. The change in actual GDP from one period to the next has a growth term and depends on (a) the gap between actual GDP and target GDP during the previous period; (b) the change in the money supply from the previous period to this period; and (c) a random error term: $Y_t - Y_{t-1} = gY_{t-1} + \alpha\left(Y_{t-1} - Y_{t-1}^*\right) + \beta\left(M_t - M_{t-1}\right) + \varepsilon_t$, where α and β are positive.
3. The change in the money supply from the previous period to this period depends on the gap between actual GDP and target GDP during the previous period: $M_t - M_{t-1} = \lambda\left(Y_{t-1} - Y_{t-1}^*\right)$, where λ is positive.
4. The criterion I would use to measure the effectiveness of the monetary policy is the root mean squared error: $\text{RMSE} = \frac{\sum\left(Y - Y^*\right)^2}{n}$. Another possibility is the mean absolute error: $\text{MAE} = \frac{\sum\left|Y - Y^*\right|}{n}$
5. A random number generator would be used to generate values of the random error term for each period of each simulation. A comparison of the RMSEs for different values of λ could be used to determine the optimal value of λ. Ideally, the analysis should be done for a variety of values of the model's other parameters in order to gauge the robustness of the results.