Econ 58
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Final Exam Answers

1a. The rate of growth of XY is approximately equal to the sum of the rates of growth of X and Y; the rate of growth of X/Y is approximately equal to the rate of growth of X minus the rate of growth of Y:

a. nominal GDP = PY implies the (approximate) rate of growth is 3% + 2% = 5%
b. the real money supply = M/P implies the (approximate) rate of growth is 4% - 3% = 1%
c. real per capita GDP = Y/L implies the (approximate) rate of growth is 2% - 1% = 1%
d. nominal per capita GDP = PY/L implies the (approximate) rate of growth is 3% + 2% - 1%= 4%

1b. The firm can sell the cheese now for \$2.00, or sell it in 12 months for P and pay \$1.20 storage costs. The present value of the net price is V = (P - \$1.20)/1.10. This is larger than \$2.00 if

$$\frac{P - \$1.20}{1.10} > \$2.00$$

$$P > \$1.20 + 1.10(\$2.00) = \$3.40$$

2. The objective function is

$$Z = 0.1K + 0.7L + \lambda\left[100 - 20K^{0.3}L^{0.7}\right]$$

The first derivatives are

$$\frac{\partial Z}{\partial K} = 0.1 - 0.3\lambda\left[20K^{-0.7}L^{0.7}\right] = 0.1 - 0.3\lambda\frac{Q}{K}$$

$$\frac{\partial Z}{\partial L} = 0.7 - 0.7\lambda\left[20K^{0.3}L^{-0.3}\right] = 0.7 - 0.7\lambda\frac{Q}{L}$$

$$\frac{\partial Z}{\partial \lambda} = -20K^{0.3}L^{0.7} + 100$$

Equating the first derivatives to 0 gives (0.3Q/K)/(0.7Q/L) = 0.1/0.7, which means the ratio of the marginal product of capital to the marginal product of labor equals the ratio of the marginal cost of capital to the marginal cost of labor. For the values here, K/L = 3.

3. The objective function is

$$Z = 20K^{0.3}L^{0.7} + \lambda(3.6 - 0.1K - 0.7L)$$

The first derivatives are

$$\frac{\partial Z}{\partial K} = 0.3\left[20K^{-0.7}L^{0.7}\right] - 0.1\lambda = 0.3\frac{Q}{K} - 0.1\lambda$$

$$\frac{\partial Z}{\partial L} = 0.7\left[20K^{0.3}L^{-0.3}\right] - 0.7\lambda = 0.7\frac{Q}{L} - 0.7\lambda$$

$$\frac{\partial Z}{\partial \lambda} = 10 - 0.1K - 0.7L$$

Equating the first derivatives to 0 gives (0.3Q/K)/(0.7Q/L) = 0.1/0.7, which means the ratio of the marginal product of capital to the marginal product of labor equals the ratio of the marginal cost of capital to the marginal cost of labor. For the values here, K/L = 3. (The duality property in production is that the inputs that maximize output for a given cost also minimize the cost of producing a given output.)

4. a. A dynamic equilibrium requires $Y = Y_{\square 1} = Y_{\square 2} = Y^e$. For this to be true, $Y^e = \square Y^e + \square Y^e$ only if $\square + \square =$ 1.
   b. Substituting $\square$ = 1 - $\square$,

$$Y = (1 \square \square)Y_{\square 1} + \square Y_{\square 2}$$

$$Y \square Y_{\square 1} = \square\square\left(Y_{\square 1} \square Y_{\square 2}\right)$$

   If $\square$ and $\square$ are positive and add to 1, then 0 < $\square$ < 1. The model is cyclically stable since the change in Y each period changes sign and is closer in absolute value to 0.

5. Revenue is PQ. The revenue-mazimizing price is given by the first-order condition:

$$0 = \frac{d(PQ)}{dP}$$

$$= Q + P\frac{dQ}{dP}$$

$$\frac{dQ}{dP}\frac{P}{Q} = \square 1$$

   Revenue is maximized when the elasticity of demand is equal to - 1. The percent change in revenue PQ is equal to the percent change in price P plus the percent change in quantity Q. If the promoter raises the price by 1% and demand falls by less than 1%, then it can increase revenue by raising the price. If the promoter raises the price by 1% and demand falls by more than 1%, then it can increase revenue by reducing the price. The price is the revenue-maximizing price when a 1% increase in the price reduces demand by 1%, leaving revenue unchanged.

6. Here are the payoff matrixes:

Firm A payoff	A high price	A low price
B high price	100	210
B low price	20	130

Firm B payoff	A high price	A low price
B high price	80	10
B low price	200	110

   The noncompetitive equilibrium is with both firms charging a low price. At A high, B high and at A high, B low, A can increase its profit by lowering its price, At A high, B high and at A low, B high, B can increase its profit by lowering its price. At A low, B low, neither firm can increase its profit by raising its price.

7. The price is maximized at

$$\frac{dP_t^e}{dt} = a \square 2bt = 0 \text{ at } t = \frac{a}{2b}$$

   However, we want to maximize the present value of the price of the asset. With continuous compounding at an interest rate R, the present value is

$$V = \frac{P_t^e}{e^{Rt}}$$

$$= \frac{P_0 + at \square bt^2}{e^{Rt}}$$

This is maximized when the rate of increase in the value of the asset is equal to the interest rate:

$$\frac{dV}{dt} = -RP_t^e e^{-Rt} + e^{-Rt}\frac{dP_t^e}{dt}$$

$$= e^{-Rt}\left[-RP_t^e + \frac{dP_t^e}{dt}\right]$$

$$= 0 \text{ when } \frac{dP_t^e}{dt}\frac{1}{P_t^e} = R$$

Substituting

$$R = \frac{dP_t^e}{dt}\frac{1}{P_t^e}$$

$$= \frac{a - 2bt}{P_0 + at - bt^2}$$

$$R\left[P_0 + at - bt^2\right] = a - 2bt$$

$$0 = a - RP_0 - (aR + 2b)t + bRt^2$$

This quadratic equation can be solved for t.

8. a. If $\beta = 0.6$, then 40% of the bees purchased leave the farm.
   b. Taking the derivative of the profit function with respect to the quantity of bees:

$$\pi = PH - C$$

$$= PA(\beta B)^{\alpha} - F - wB$$

$$\frac{d\pi}{dB} = \alpha PA\beta^{\alpha}B^{\alpha-1} - w$$

$$= 0 \text{ if}$$

$$B = \left[\frac{\alpha PA\beta^{\alpha}}{w}\right]^{\frac{1}{1-\alpha}}$$

c. B would clearly be higher if β were larger. Mathematically,

$$B = \left[\frac{\alpha PA\beta^{\alpha}}{w}\right]^{\frac{1}{1-\alpha}}$$

$$= \left[\frac{\alpha PA}{w}\right]^{\frac{1}{1-\alpha}}\beta^{\frac{\alpha}{1-\alpha}}$$

$$\frac{dB}{d\beta} = \left[\frac{\alpha PA}{w}\right]^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{1-\alpha}\right]\beta^{\frac{\alpha}{1-\alpha}-1} > 0$$

d. This is a maximum because the second derivative of the profit function is negative:

$$\frac{d\pi}{dB} = \alpha P A^{\beta} B^{\alpha-1} - w$$

$$\frac{d^2\pi}{dB^2} = \alpha(\alpha-1) P A^{\beta} B^{\alpha-2} < 0$$

e. The more bees stay on the farm, the more incentive there is to acquire bees.

9-10. Two labor markets are enough to capture these features. Let the demand for labor in each market depend on the real wage, with a stochastic term to incorporate fluctuations in labor demand:

$$D_1 = a_1 - b_1 \frac{W_1}{P_1} + \varepsilon_1$$

$$D_2 = a_2 - b_2 \frac{W_2}{P_2} + \varepsilon_2$$

Let the supply of labor in each market depend on wages in the two markets

$$S_1 = c_1 + c_1 \frac{W_1}{W_2}$$

$$S_2 = d_2 + d_2 \frac{W_2}{W_1}$$

Let the adjustment of wages depend on the difference between labor demand and supply

$$\Delta W_1 = \lambda_1 (D_1 - S_1)$$

$$\Delta W_2 = \lambda_2 (D_2 - S_2)$$

Let employment in each market equal the minimum of labor demand and supply.

$$E_1 = \min(D_1, S_1)$$

$$E_2 = \min(D_2, S_2)$$