Econ 58
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Final Exam Answers

1. If we substitute the equations for c, i, and g into y = c + i + g, we can solve for y:

$$y = 300 + 0.6y + i_0 + g_0$$
$$= \frac{300 + i_0 + g_0}{1 - 0.6}$$

   a. The comparative-static effect of an increase in i_0 on y is $\partial y/\partial i_0 = 1/(1 - 0.6) = 2.5$.
   b. I expect the effect to be larger because the increase in y caused by the increase in investment will now also increase government spending, further increasing y.
   c. If g = 0.1y, then

$$y = 300 + 0.6y + i_0 + 0.1y$$
$$= \frac{300 + i_0}{1 - 0.7}$$

   Now the comparative-statics effect is larger: $\partial y/\partial i_0 = 1/(1 - 0.7) = 3.3$. This is an example of the general macroeconomic principle that the effects of a change in spending on the economy are stronger if spending is more sensitive to changes in y.

2. a. The elasticity of demand is

$$\varepsilon = \left|\frac{dQ}{dP}\frac{P}{Q}\right|$$
$$= \left|-b\frac{P}{Q}\right|$$
$$= \frac{bP}{Q}$$
$$= \frac{bP}{a - bP}$$

   b. The elasticity of demand can be larger or smaller than 1.

   c. The elasticity of demand increases as P increases:

$$\frac{d\varepsilon}{dP} = \frac{(a - bP)b + bPb}{(a - bP)^2}$$
$$= \frac{a}{(a - bP)^2}$$
$$> 0$$

3. The present value of the firm's profit is

$$V = R_1 + \frac{R_2}{1 + r}$$

The firm wants to maximize this Lagrangian:

$$Z = R_1\left[Q_1\right] + \frac{R_2\left[Q_2\right]}{1+r} + \lambda\left(100 - Q_1 - Q_2\right)$$

The first-order conditions are

$$0 = \frac{\partial Z}{\partial Q_1} = \frac{dR_1}{dQ_1} - \lambda$$

$$0 = \frac{\partial Z}{\partial Q_2} = \frac{1}{1+R}\frac{dR_2}{dQ_2} - \lambda$$

Solving:

$$\frac{dR_1}{dQ_1} = \frac{1}{1+R}\frac{dR_2}{dQ_2}$$

Because one more unit sold today means one less unit sold a year from now, the present value of the increased revenue from selling one more unit today should equal the present value of selling one more (or one less) unit tomorrow.

4. The objective function is

$$Z = 1 + 0.3\sqrt{X} + 0.4\sqrt{Y} + \lambda\left(25 - X - Y\right)$$

The first derivatives are

$$\frac{\partial Z}{\partial X} = 0.5\left(0.3\right)X^{-1/2} - \lambda$$

$$\frac{\partial Z}{\partial R} = 0.5\left(0.4\right)Y^{-1/2} - \lambda$$

$$\frac{\partial Z}{\partial \lambda} = X + Y - 25$$

Equating the first derivatives to 0 gives X/Y = 9/16, so that X = 9, Y = 16. GPA is maximized when the ratio of the marginal effect on GPA of studying more for the first course to the marginal effect on GPA of studying more for the second course equals the marginal cost of studying more for the first course in terms of studying for the second course (1 since one hour more for one course means one hour less for the other course):

$$\frac{\partial GPA / \partial X}{\partial GPA / \partial Y} = \text{cost of } X \text{ in terms of } Y$$

$$\frac{0.15X^{-1/2}}{0.20Y^{-1/2}} = 1$$

$$\frac{Y^{1/2}}{X^{1/2}} = \frac{4}{3}$$

$$\frac{Y}{X} = \frac{16}{9}$$

5. We can express the level of the government debt as a function of the previous period debt and the current surplus:

$$B_t = (1+R)B_{t-1} - S_t$$

Now starting at period t = 0:

$$B_1 = (1+R)B_0 - S_1$$
$$B_2 = (1+R)B_1 - S_2$$
$$= (1+R)^2 B_0 - (1+R)S_1 - S_2$$

Continuing,

$$B_n = (1+R)^n B_0 - (1+R)^{n-1} S_1 - (1+R)^{n-2} S_2 - \ldots S_n$$

Setting $B_n = 0$ and dividing by $(1+R)^n$, we have the condition that the present value of the future surpluses must equal the current level of debt:

$$B_0 = \frac{S_1}{(1+R)^1} + \frac{S_2}{(1+R)^2} + \ldots + \frac{S_n}{(1+R)^n}$$

6. a. The dynamic equilibrium wage W^e is where wages are constant, $W_{t+1} = W_t = W^e$:

$$W^e - W^e = \lambda\left(\frac{a-c}{b+d} - W^e\right)$$

$$W^e = \frac{a-c}{b+d}$$

b. This makes sense because demand is equal to supply when $W = (a - c)/(b + d)$:

$$D_t = S_t$$
$$a - bW_t = c + dW_t$$
$$W_t = \frac{a-c}{b+d}$$

c. To investigate dynamic stability,

$$W_{t+1} - W_t = \lambda\left(\frac{a-c}{b+d} - W_t\right)$$

$$W_{t+1} = \lambda\left(\frac{a-c}{b+d}\right) + (1-\lambda)W_t$$

$$W^e = \lambda\left(\frac{a-c}{b+d}\right) + (1-\lambda)W^e$$

$$\left(W_{t+1} - W^e\right) = (1-\lambda)\left(W_t - W^e\right)$$

The model is monotonically stable if $0 < \lambda < 1$

d. The model is cyclically stable if $1 < \lambda < 2$

e. The conditions in Parts (c) and (d) make sense. We can write the dynamic equation like this:

$$W_{t+1} - W_t = \lambda\left(W^e - W_t\right)$$

If $0 < \lambda < 1$, then the wage is adjusting part way toward the equilibrium wage. If $1 < \lambda < 2$, then the wage is overshooting equilibrium, but the gap between the wage and the equilibrium wage is shrinking.

7. The objective function is

$$Z = U\left[A_1, O_1\right] + \lambda_1\left(30 - V\left[A_2, O_2\right]\right) + \lambda_2\left(20 - A_1 - A_2\right) + \lambda_3\left(30 - O_1 - O_2\right)$$

The first derivatives are

$$\frac{\partial Z}{\partial A_1} = \frac{\partial U}{\partial A_1} - \lambda_2$$
$$\frac{\partial Z}{\partial O_1} = \frac{\partial U}{\partial O_1} - \lambda_3$$
$$\frac{\partial Z}{\partial A_2} = -\lambda_1 \frac{\partial V}{\partial A_2} - \lambda_2$$
$$\frac{\partial Z}{\partial O_2} = -\lambda_1 \frac{\partial V}{\partial O_2} - \lambda_3$$

Setting these first derivatives equal to zero and taking the ratios of the first equation to the second equation, and the ratio of the third equation to the fourth equation, we have

$$\frac{\partial U / \partial A_1}{\partial U / \partial O_1} = \frac{\lambda_2}{\lambda_3}$$
$$\frac{\partial V / \partial A_1}{\partial V / \partial O_1} = \frac{\lambda_2}{\lambda_3}$$

so that the ratio of Claire's marginal utilities is equal to the ratio of Cory's marginal utilities

$$\frac{\partial U / \partial A_1}{\partial U / \partial O_1} = \frac{\partial V / \partial A_1}{\partial V / \partial O_1}$$

That is, the slopes of their indifference curves are tangent so that the terms at which Claire is willing to trade apples for oranges is equal to the terms at which Cory is willing to trade apples for oranges. If this were not the case, there would be room for a deal. (We can also draw an Edgeworth Box to illustrate this.)

8. If A shows 1 with probability p, 2 with probability q, and 3 with probability 1 - p - q, the expected value of A's payoff is

$$\begin{aligned} A &= 1p(1/3) - 1p(1/3) + 1p(1/3) - 1q(1/3) + 1q(1/3) - 1q(1/3) \\ &\quad + 1(1-p-q)(1/3) - 1(1-p-q)(1/3) + 1(1-p-q)(1/3) \\ &= p(1/3) - q(1/3) + (1-p-q)(1/3) \\ &= (1/3)(1-2q) \end{aligned}$$

This is maximized with q = 0 and any value for p; that is, don't show 2 fingers.

9. Use a contingency table with 100,000 homes:

	fire	no fire	total
careless	10	990	1,000
careful	99	98,901	99,000
total	109	99,891	100,000

The probability that a home destroyed by fire was occupied by a careless person is 10/109: 0.092. This 9.2% figure is much larger than the 1% of the total population that is careless, but it is still far from certain that a house destroyed by fire was occupied by a careless household.

10. If we let L_1 be the amount of low-skill labor and L_2 the amount of high-skill labor, then this Cobb-Douglas production function satisfies requirements (a) and (b) above.

$$Q = AL_1^{\alpha}L_2^{\beta} \quad , \quad 0 < \alpha < \beta < 1$$

Letting the two types of wages be W_1 and W_2, and allowing fixed costs B, a firm's profits are

$$\pi = PQ - \left(B + W_1L_1 + W_2L_2\right)$$

$$= PAL_1^{\alpha}L_2^{\beta} - B - W_1L_1 - W_2L_2$$

If firms are competitive price takers, as in Requirement (c), then profit maximization implies

$$0 = \frac{\partial \pi}{\partial L_1} = \alpha P \frac{Q}{L_1} - W_1$$

$$0 = \frac{\partial \pi}{\partial L_2} = \beta P \frac{Q}{L_2} - W_2$$

Taking the ratio of these two equations,

$$\frac{\alpha P \frac{Q}{L_1}}{\beta P \frac{Q}{L_2}} = \frac{W_1}{W_2}$$

$$\frac{\alpha L_2}{\beta L_1} = \frac{W_1}{W_2}$$

Because every firm has the same production function and faces the same wages, every firm has the same ratio of low-skill workers to high-skill workers. With requirement (d), the ratio of low-skill workers to high-skill workers is given.

a. If there are equal amounts of both kinds of labor, $L_1 = L_2$, then the wages of the low-skill workers are lower than the wages of high-skill workers:

$$\frac{W_1}{W_2} = \frac{\alpha L_2}{\beta L_1}$$

$$= \frac{\alpha}{\beta} < 1 \text{ if } \alpha < \beta$$

b. If there is an increase in the supply of low-skill workers relative to high-skill workers, the ratio of low-skill wages to high-skill wages will decrease:

$$\frac{W_1}{W_2} = \frac{\alpha L_2}{\beta L_1} = \frac{\alpha}{\beta}\left(\frac{L_1}{L_2}\right)^{-1}$$

$$\frac{\partial\left(\frac{W_1}{W_2}\right)}{\partial\left(\frac{L_1}{L_2}\right)} = -\frac{\alpha}{\beta}\left(\frac{L_1}{L_2}\right)^{-2} < 0$$