Econ 58
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Final Exam (150 minutes)
Write your answers directly on the exam. No calculators allowed.

1. Consider a firm whose output Q is related to its labor input L by the following production function:

$$Q = AL^{2/3}$$

This firm wants to maximize its profits $\Pi = PQ - (B + WL)$, where P is the fixed price it receives for its output, B is the fixed cost of producing output, and W is the fixed wage rate it pays its workers.

a. Determine the profit-maximizing level of L, as a function of A, B, W, and P.

b. Show that this is the profit-maximizing, not profit-minimizing, solution.

c. Are profits positive or negative at this profit-maximizing level of L?

d. Is the profit-maximizing level of L related positively or negatively to B? Explain why.

e. Is the profit-maximizing level of L related positively or negatively to P? Explain why.

2. (Continuation of question 1)

f. Rewrite the equation in part (a) to show how W/P is related to A, B, and L.

g. What is the economic interpretation of the right-hand side of the equation in part (f)?

h. Prove that the relationship between W/P and L in part (f) is positively or negatively sloped.

i. Prove that the relationship between W/P and L in part (f) is curved upward or downward.

j. Sketch the relationship between W/P (vertical axis) and the profit-maximizing level of L (horizontal axis).

3. In August 2001, a Powerball lottery had 4 jackpot winners. Each winner had the choice of an immediate payment of $41 million or an annuity that will pay $2.9 million a year for 25 years, beginning immediately. For what interest rate is the present value of the annuity equal to $41 million? (You do not need to give an explicit answer, but you do need to show the equation used to determine the answer.)

4. Consider the following two-country income-expenditure model

$$y_1 = c_1 + i_1 + g_1 + x_1 - m_1 \qquad y_2 = c_2 + i_2 + g_2 + x_2 - m_2$$

$$c_1 + i_1 + g_1 = a_1 + b_1 y_1 \quad , \quad 0 < b_1 < 1 \qquad c_2 + i_2 + g_2 = a_2 + b_2 y_2 \quad , \quad 0 < b_2 < 1$$

$$x_1 = m_2 \qquad x_2 = m_1$$

$$m_1 = e_1 y_1 \quad , \quad 0 < e_1 < b_1 \qquad m_2 = e_2 y_2 \quad , \quad 0 < e_2 < b_2$$

where the endogenous variables are y_1, c_1, i_1, g_1, x_1, m_1, y_2, c_2, i_2, g_2, x_2, and m_2, and the exogenous parameters are a_1, b_1, e_1, a_2, b_2, e_2.

a. Determine the comparative-static multiplier $\frac{\partial y_1}{\partial a_1}$

b. Explain why you believe $\frac{\partial y_1}{\partial a_1}$ would be larger or smaller if $e_1 = e_2 = 0$.

c. Show that your intuition in Part (b) is correct.

5. Consider the following dynamic model of demand and supply:

$$S_t = -a + bP_t + cP_{t-1}$$
$$D_t = d - eP_t$$
$$D_t = S_t$$

where the parameters a, b, c, d, and e are all positive. Is this model dynamically stable?

6. A Pomona student was asked this question in a banking job interview. A can contains 20 coins, 19 of which are normal coins and 1 of which has heads on both sides. A coin is randomly selected from the can and flipped five times. It lands heads all five times. What is the probability that it is the two-headed coin? (Hint: the probability that a normal coin will land heads five times in five flips is 1/32.)

7. A nation's net exports B is equal to its exports X minus its imports M:

$$B = X - eM$$

Because imports are valued in terms on the foreign currency, they are multiplied by the exchange rate e (the amount of domestic currency needed to buy one unit of the foreign currency). Exports are positively related to the exchange rate and imports are negatively related to the exchange rate:

$$X = X[e], \quad dX/de > 0$$

$$M = M[e], \quad dM/de < 0$$

a. Determine the total effect on net exports B of a change in the exchange rate e: dB/de.

b. If it is initially true that B = 0 (i.e., X = eM), what must be true of the export and import elasticities

$$\left|\frac{dX}{de}\frac{e}{X}\right| \text{ and } \left|\frac{dM}{de}\frac{e}{M}\right|$$

for dB/de to be positive?

8. In the constant dividend-growth model, the value of a stock that currently pays a dividend D, whose dividend will grow at an annual rate g, and whose dividends are discounted by a required rate of return R, is

$$V = \frac{D}{R - g}$$

a. Determine the elasticity of value with respect to R: $\varepsilon = \left|\frac{\partial V}{\partial R}\frac{R}{V}\right|$

b. Determine whether this elasticity ε is higher or lower for growth stocks; i.e., whether this elasticity is increased or decreased by an increase in g.

9. A soccer player has a penalty kick. She is certain to score if she aims right and the goalie jumps left, or if she aims left and the goalie jumps right. If she aims right and the goalie jumps right, she has a 20% chance of scoring; if she aims left and the goalie jumps left, she has a 30% chance of scoring. Assume that both players are rational and choose optimal strategies based on the assumption that the other person also chooses her optimal strategy. What is her optimal strategy? What is the goalie's optimal strategy?

10. Suppose that a fraction ω_1 of wealth is invested in a risky asset whose return has a mean μ_1 and standard deviation σ_1, a fraction ω_2 is invested in a risky asset whose return has a mean μ_2 and standard deviation σ_2, and that the returns on the two risky assets are perfectly negatively correlated. The variance of the portfolio return is

$$\sigma^2 = \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 - 2\omega_1\omega_2\sigma_1\sigma_2 \quad , \quad 0 < \omega_1 < 1,\ 0 < \omega_1 < 1,\ \omega_1 + \omega_2 = 1$$

Now suppose that $\sigma_1 = \sigma_2$.

a. Use the Lagrangian method to determine the formula for the value of ω_1 that minimizes σ^2.

b. Show that the solution in Part (a) is a minimum.

c. Determine the minimum value of σ^2.