Econ 58
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Final Exam (150 minutes)
Write your answers directly on the exam. No calculators allowed.

1. Here are some U. S. economic data:

	2003	2004
average hourly wage	$17.75	$18.09
consumer price index	183.9	189.4
interest rate on 1-year Treasury bonds	1.10%	2.11%

Write the formulas for calculating the real magnitudes of the following variables. Do not calculate the values; simply write the formulas needed for the calculations.

a. The exact percentage change in nominal wages between 2003 and 2004:

b. The exact percentage change in real wages between 2003 and 2004:

c. Either the exact or approximate real interest rate in 2003:

2. A nation's production possibility frontier (PPF) describing the efficient levels of production of X and Y is

$$X^2 + Y^2 = C \quad , C > 0$$

If the PPF is drawn with X is on the horizontal axis and Y is on the vertical axis,

a. Determine the equation for the slope $\frac{dY}{dX}$; is the slope positive, negative, or zero?

b. Does the slope increase, decrease, or stay the same as X increases? Show your work.

c. Draw a sketch of this PPF for C = 50. Be sure to label your axes.

3. Now suppose that the utility function for the country in Exercise 2 is

$$U[X, Y] = AX^{\alpha}Y^{\beta}$$

where A, α, and β are all positive parameters. If an indifference curve is drawn with X on the horizontal axis and Y is on the vertical axis,

a. Determine the equation for the slope $\frac{dY}{dX}$; is the slope positive, negative, or zero?

b. Does the slope increase, decrease, or stay the same as X increases? Show your work.

c. Draw a sketch of this indifference curve for $\alpha = \beta = 0.5$.

4. a. Use the Lagrangian method to determine the value of the ratio Y/X that maximizes the utility function given in Exercise 3 subject to the PPF constraint given in Exercise 2.

b. Draw a sketch of the PPF and indifference curve when utility is maximized.

5. Suppose that the country described in Exercises 2-4 can trade X for Y with other countries on these terms:

$$P_X X_C + P_Y Y_C = P_X X + P_Y Y$$

where X_C and Y_C are the amounts that the country consumes, X and Y are the amounts it produces, and P_X and P_Y are the world prices; we consequently rewrite the country's utility function as

$$U[X_C, Y_C] = A X_C^{\alpha} Y_C^{\beta}$$

Use the Lagrangian method to determine the values of the ratios Y/X and Y_C/X_C that maximize this utility function given the country's PPF and available terms of trade.

6. A fish ranch can buy baby fish for \$C each, raise the fish, and then sell each fish after t years for a price

$$P = Ce^{\alpha t - \beta t^2}$$

a. What is the optimal harvest time t?

b. Explain why, at the optimal harvest time, the percentage increase in the value of the fish as time passes is larger than, smaller than, or equal to the interest rate.

7. Two firms are bidding for a potentially lucrative government contract. Each firm must submit its bid (one bid per firm) in a sealed envelope. When the government agency opens the envelopes, the contract is awarded to the firm with the highest bid. Here is a stylized payoff matrix with the first number in each cell being the payoff to Firm 1 and the second number the payoff to Firm 2. (These numbers reflect the assumptions that if each firm submits a low bid or each submits a high bid, there is a 50% chance of receiving the contract.)

		Firm 2	
		low	high
Firm 1	low	50, 50	0, 75
	high	75, 0	-25, -25

a. Is there a noncooperative equilibrium?

b. If Firm 1 chooses a low bid with probability p and Firm 2 chooses a low bid with probability q, and each firm maximizes the expected value of its own payoff, what are the equilibrium values of p and q?

8. Consider this dynamic income-expenditure model:

$$Y = C + I + G$$
$$C = c_0 + c_y(Y_{-1} - T_{-1}) \qquad 0 < c_y$$
$$I = i_0$$
$$G = g_0$$
$$T = t_y Y \qquad 0 < t_y < 1$$

where Y is national income, C is consumption spending, I is investment spending, G is government spending, and T is taxes. This model's parameters are c_0, c_y, i_0, g_0, and t_y. Notice that the marginal propensity to spend need not be less than 1. Is this model more likely to be dynamically stable if t_y is large or small? Derive your answer mathematically and also explain the economic logic without using any mathematical symbols.

9. The demands for two products are given by these two equations relating the two demands D to the two prices P:

$$D_1 = d_{10} + d_{11}P_1 + d_{12}P_2 \ , \ d_{11} < 0, \ d_{12} > 0$$
$$D_2 = d_{20} + d_{21}P_1 + d_{22}P_2 \ , \ d_{21} > 0, \ d_{22} < 0$$

The supplies are fixed at S_1 and S_2 respectively. If we assume that demand is equal to supply in each market, for what values of the parameters is there no unique solution for the equilibrium prices?

10. Your assignment is to create a model that is simple, but not simplistic, that can be used to estimate (a) the median value of Pomona College's endowment 50 years from now; and (b) the probability that, sometime in the next 50 years, the real value of the endowment will fall below 50% of the current value. Each year, the endowment earns an uncertain rate of return; is augmented by donations from graduates and others; and is diminished by withdrawals used to pay part of the cost of running the college. Be sure to: (a) define all variables, (b) briefly explain how you would determine the values of the model's parameters, and (c) explain how the model can be used to answer the questions posed.