Final Exam (150 minutes)

Write your answers directly on the exam. No calculators allowed.

1. When you graduate from Pomona College, your annual salary will be Y_0.
   a. Write down an equation for determining your annual salary t years after graduation if your salary grows at a continuously compounded growth rate of 5%.

   b. If you spend 90% of your salary every year, what is the continuously compounded rate of growth of your spending?

   c. If you save 10% of your salary every year, what is the continuously compounded rate of growth of your saving?

2. Suppose that an individual's utility U depends on consumption C and leisure R, and that this person can earn a real wage of 10 per hour and can work from L = 0 to 24 hours a day, so that C = 10L:

$$U = 6(10L)^{1/4}R^{2/4}$$

$$24 = L + R$$

Use the Lagrangian method to determine the amount of work L that maximizes this person's utility.

3. Consider this income-expenditure model

$$Y = E$$

$$E = a + bY + cY_{-1}$$

where Y is aggregate income, E is aggregate spending, and a, b, and c are positive parameters.

a. What is the comparative-static effect on dynamic equilibrium income of a change in the parameter a?

For each of the following questions, give the general answer and then give an example of specific parameter values. (For example, your answer might be something like this: "$b < c$; for example, $b = 0.3$, $c = 0.4$.")

b. Under what conditions is the model monotonically stable?

c. Under what conditions is the model monotonically unstable?

d. Under what conditions is the model cyclically stable?

e. Under what conditions is the model cyclically unstable?

4. In this stylized example, there are two candidates for governor, and a fraction π of the voters favor Candidate A and a fraction $1 - \pi$ favor Candidate B. The value of π is initially considered equally likely to be 0.49 or 0.51: $P[\pi = 0.49] = P[\pi = 0.51] = 1/2$. (For simplicity, we assume that no other values of π are possible.) Now, a random sample of 1000 voters finds that 505 will vote for A and 495 will vote for B. The binomial distribution tells us the probability of this poll result if $\pi = 0.49$ and if $\pi = 0.51$:

$$P\left[\frac{x}{n} = \frac{505}{1000} \text{ if } \pi = 0.49\right] = \binom{1000}{505} 0.49^{505} 0.51^{495} = 0.016084$$

$$P\left[\frac{x}{n} = \frac{505}{1000} \text{ if } \pi = 0.51\right] = \binom{1000}{505} 0.51^{505} 0.49^{495} = 0.023996$$

What are the revised probabilities that $\pi = 0.49$ and $\pi = 0.51$?

5. It is the last play of the football game and SportsU needs a touchdown to win. If SportsU tries a running play, SportsU has a 30% chance of winning if their opponent chooses a running defense and an 80% chance of winning if their opponent choosing a passing defense. If SportsU tries a passing play, SportsU has a 20% chance of winning if their opponent chooses a passing defense and an 90% chance of winning if their opponent choosing a running defense. SportsU wants to maximize its probability of winning; its opponent wants to maximize its own chances of winning.
   a. Is there a noncooperative equilibrium if each team chooses a fixed strategy?

   b. If SportsU has a probability p of choosing a running play and its opponent has a probability q of choosing a running defense, what are the optimal values of p and q?

6. Who pays the 6% real estate commission, the buyer or seller? Conventionally, home sellers pay their real estate agent a commission equal to 6% of the sale price, with half of this commission going to the seller's realtor and half to the buyer's realtor. For example, if a home sells for $500,000, the seller pays a $30,000 commission, of which $15,000 goes to the seller's agent and $15,000 to the buyer's agent.

   Suppose that the demand D and supply S for comparable homes depend on the price P as follows:

$$D = AP^{-\alpha}$$

$$S = B(\lambda P)^{\beta}$$

   where all of the parameters are positive and λ is equal to 1 minus the real estate commission; for example, $\lambda = 1 - 0.06 = 0.94$ if there is a 6% commission.

   a. What is the elasticity of demand with respect to price?

   b. What is the elasticity of supply with respect to price?

   c. Explain the logic behind writing the supply equation as $S = B(\lambda P)^{\beta}$, as opposed to $S = BP^{\beta} - \lambda$

   d. What is the market equilibrium price, where demand is equal to supply?

7. Continuing the preceding exercise

   a. What is the elasticity of the market equilibrium price with respect to λ?

   b. What is the value of the elasticity of the market equilibrium price with respect to λ if $\beta = 0$?

   c. Explain the economic reasoning behind your answer to part (b), using words not math.

8. In this IS-LM model, M_0 is the exogenous money supply, Y is endogenous income, R is the endogenous interest rate, and the other terms are all exogenous parameters:

$$Y = E_0 + E_Y Y + E_R R$$

$$M_0 = L_0 + L_Y Y + L_R R$$

a. Write this model in matrix form:

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b. For what values of the parameters does this model not have a unique solution?

c. Explain, in ordinary English, how a model with 2 equations and 2 endogenous variables could have no unique solution.

9. Suppose that the initial cost of planting a stand of trees is C = 50 and the value of the volume (in cubic meters) of marketable timber t years after planting is

$$V[t] = 100t^{0.9}$$

A stand will be planted immediately and harvested after t_1 years, with a new stand then immediately planted and harvested after t_2 additional years; no further stands will be planted. Set up a model that can be used to determine the optimal values of t_1 and t_2. You do not need to solve the model; you do need to write down an explicit model that can be used to determine the optimal harvest times.

For example, your answer might be something like this: "The total value is $W[t] = 100(t_1 + t_2)^{0.9}$. The optimal harvest times can be determined by taking the partial derivatives of W with respect to t_1 and t_2 and equating them: $\frac{\partial W}{\partial t_1} = \frac{\partial W}{\partial t_2}$."

10. Your assignment is to build a model to gauge the effectiveness of an activist monetary policy. The model should have the following features:
    1. Target GDP grows at a constant rate.
    2. The change in actual GDP from one period to the next depends on: (a) the gap between actual GDP and target GDP during the previous period; (b) the change in the money supply from the previous period to this period; and (c) a random error term.
    3. The change in the money supply from the previous period to this period depends on the gap between actual GDP and target GDP during the previous period.

    You need to
    a. write down explicit equations for your model.
    b. specify the signs of all the parameters.
    c. explain the criterion you would use to measure the effectiveness of the monetary policy.
    d. explain how you would use the model to determine the effectiveness of monetary policy, using the criterion you describe in (c).