Final Exam (150 minutes)

Write your answers directly on the exam. No calculators allowed. ALWAYS show your work. Each of the 10 questions is worth 10 points. Notice that the first question has two parts (each worth 5 points) and the last exercise is Questions 9 and 10 (worth a total of 20 points).

1a. The following variables might appear in a macroeconomic growth model:

Y: real GDP
M: money supply
P: price level
L: labor supply

Suppose that Y is growing by 2% per year, M is growing by 4% per year, P is growing by 3% per year, and L is growing by 1% per year. What are approximate annual growth rates of

a. nominal GDP

b. the real money supply

c. real per capita GDP

d. nominal per capita GDP

1b. Suppose that fresh cheddar cheese can be sold for $2.00 a pound that and that it costs $0.10/month to store cheese for aging. If the annual interest rate is 10%, what would the price of sharp cheese (aged 12 months) have to be to make it worthwhile to age this cheese for 12 months? (Assume that the storage costs are paid at the end of the storage period; that is, $1.20 at the end of 12 months.)

2. Suppose that a firm's output is characterized by this production function

$$Q = 20K^{0.3}L^{0.7}$$

where K and L are the amounts of capital and labor employed and Q is the amount produced. The cost of employing capital and labor is given by this equation

$$C = 0.1K + 0.7L$$

where 0.1 is capital's rental rate and 0.7 is labor's real wage rate. Use the Lagrangian method to determine the capital-labor ratio K/L that minimizes the cost C of producing output Q = 100.

3. Using the production function in the preceding exercise, use the Lagrangian method to determine the capital-labor ratio K/L that maximizes the output that can be produced at a total cost C = 3.6.

4. Consider the following dynamic model of real GDP:

$$Y = \square Y_{\sqcap 1} + \square Y_{\sqcap 2}$$

where □ and □ are positive parameters.

   a. What must be true of □ and □ for there to exist a dynamic equilibrium?
   b. If the condition in part (a) is true, is this model monotonically stable, cyclically stable, monotonically unstable, or cyclically unstable?

5. A concert promoter can sell up to Q = 50,000 tickets to an upcoming performance; seating is first-come, first-served. If the promoter sets the ticket price P to maximize revenue, what will be true of the elasticity of demand $\frac{dQ}{dP}\frac{P}{Q}$ at this revenue-maximizing price? Explain the logic behind this condition in ordinary English.

6. In this stylized model, two firms, A and B, sell a very similar product, and each can choose to charge either a high price or a low price. If both firms charge a high price, Firm A's profit will be 100 and Firm B's profit will be 80; if Firm A charges a high price and Firm B charges a low price, Firm A's profit will be 20 and Firm B's profit will be 200; if Firm B charges a high price and Firm A charges a low price, Firm A's profit will be 210 and Firm B's profit will be 10; and if both firms charge a low price, Firm A's profit will be 130 and Firm B's profit will be 110. Show that there is or is not a noncooperative equilibrium.

7. You believe that we are in a speculative bubble and your expectations about the price of an asset are given by this equation:

$$P_t^e = P_0 + at - bt^2 \quad , \quad t < \frac{a}{b}$$

where a and b are both positive parameters. This asset does not pay any cash before you sell it. When is the optimal time to sell this asset if your price expectations are correct? You will not be able to obtain a numerical solution since you do not know all the parameter values; however, you can show the method used to obtain the solution. Also explain in ordinary English the nature of this solution and the logic behind it.

8. Consider this model of a honey farm. The price P of honey is fixed and the amount of honey produced H is related to the number of bees B purchased:

$$H = A(\square B)^{\square}$$

where $A > 0$, $0 < \square < 1$, and $0 < \square < 1$. The parameter □ reflects the reality that some of the bees purchased will leave the farm and populate hives elsewhere. The cost C of running the honey farm is equal to a fixed cost F plus the cost w per bee of acquiring the bees:

$$C = F + wB$$

a. What does it mean in plain English if $\square = 0.6$?

b. Find the profit-maximizing quantity of bees.

c. Show that the profit-maximizing quantity of bees would be either higher or lower if □ were larger.

d. Show that you answer to (c) is a maximum, not a minimum

e. Explain your answer to (c) in ordinary English

9-10. James Tobin has argued that the essence of the labor market is that there are multiple markets in constant flux. Specify an appealing short-run model of labor markets that has these characteristics:

a. There is more than one labor market.
b. The demand for labor in each market fluctuates.
c. Workers have some mobility; for example, construction workers in Detroit can decide to move to take jobs in Phoenix.
d. Wages respond to labor demand and supply, but do not instantaneously equate demand and supply.
e. Employment depends on labor demand and supply.

You do not have to solve your model, but you MUST specify the model's equations—words cannot be used in place of equations. You may find that graphs help your reasoning but, again, graphs cannot be used in place of equations. Your grade will depend not on the number of equations in the model, but on how realistically your equations incorporate the features enumerated above. You should not model aggregate demand and supply; stick to the labor market.