Econ 58
Gary Smith
Spring 2006

Final Exam (150 minutes)
Write your answers directly on the exam. No calculators allowed. ALWAYS show your work.
Each of the 10 questions is worth 10 points.

1. In the following income-expenditure model, income y is equal to the sum of private consumption c, private investment i, and government spending g:

$$y = c + i + g$$
$$c = 300 + 0.6y$$

Investment i is exogenous: $i = i_0$.

a. Determine the value of the comparative-static effect of an increase in i_0 on y if government spending is exogenous, $g = g_0$.

b. Explain why, without using any math, you think that the comparative-static effect of an increase in i_0 on y would be larger or smaller if government spending depended on y: $g = 0.1y$.

c. Determine the value of the comparative-static effect in Part (b), and see if your intuition is correct.

2. Consider a linear demand function:

$$Q = a - bP, \;\; a > 0, \;\; b > 0$$

a. Derive the formula for the elasticity of demand $\epsilon = \left| \frac{dQ}{dP} \frac{P}{Q} \right|$

b. Is the elasticity of demand larger or smaller than 1?

c. Show that the elasticity of demand increases, decreases, or stays the same as P increases.

3. A firm has a total supply of a natural resource, $Q = 100$, of which they will sell Q_1 now and the remainder, $Q_2 = 100 - Q_1$, a year from now. Their revenue will be $R_1[Q_1]$ and $R_2[Q_2]$. There are no costs of producing or selling this resource. Derive a general optimal rule for this firm and state this rule in ordinary English; for example, the elasticities of revenue should add to 1: $\frac{dR_1}{dQ_1} \frac{Q_1}{R_1} + \frac{dR_1}{dQ_1} \frac{Q_1}{R_1} = 1.$

4. A student has 25 hours available to study for two final exams. Here is how this student's grade point average (GPA) for these courses is related to the time spent studying for these finals:

$$G = 1 + 0.3\sqrt{X} + 0.4\sqrt{Y}$$

$$X + Y = 25$$

Where G is the GPA for these two courses, X is the number of hours spent studying for the first course, and Y is is the number of hours spent studying for the second course. Use the Lagrangian method to determine the value of X that maximizes GPA.

5. The change in a government's outstanding debt B is equal to the interest on its debt minus the government surplus S (tax revenue net of government outlays other than debt interest):

$$B_t - B_{t-1} = RB_{t-1} - S_t$$

where R is the (constant) interest rate on its debt. The current government debt is B_0. In order for government debt at some future date to equal to zero ($B_n = 0$), what must be true of government surpluses between now ($t = 0$) and period $t = n$? Derive this condition mathematically and then state this condition in ordinary English.

6. Consider the following model of wage adjustment in the labor market:

$$D_t = a - bW_t$$

$$S_t = c + dW_t$$

$$W_{t+1} - W_t = \lambda\left[\frac{a - c}{b + d} - W_t\right]$$

where D is labor demand, S is labor supply, W is the wage rate, and b, d, and λ are positive parameters.

a. What is the dynamic equilibrium wage rate?

b. Is labor demand larger than, less than, or equal to labor supply related at this dynamic equilibrium wage rate?

c. Under what conditions is this model monotonically stable?

d. Under what conditions is this model cyclically stable?

e. Explain why the conditions in Parts (c) and (d) make sense logically.

7. Claire wants to maximize her utility $U[A_1, O_1]$ and Cory wants to maximize his utility $V[A_2, O_2]$, where A_1 and O_1 are the quantities of apples and oranges received by Claire and A_2 and O_2 are the quantities received by Cory. There are fixed amounts of apples and oranges available:

$$20 = A_1 + A_2$$
$$30 = O_1 + O_2$$

Set up the equations you would use to determine the allocation of apples and oranges that maximizes Claire's utility for a given level of utility for Cory. For example, if Cory's utility is to be 30, how can we allocate apples and oranges between Claire and Cory so as to keep Cory's utility at 30 and maximize Claire's utility? At this Pareto Optimal position, is the ratio of Claire's marginal utility of apples to her marginal utility of oranges larger than, smaller than, or equal to the ratio of Cory's marginal utility of apples to his marginal utility of oranges? Show this mathematically and then explain it logically.

8. In the Indian game Tong, two players, A and B, simultaneously show 1, 2 or 3 fingers, with A winning if the sum is even and B winning if the sum is odd. A’s payoff matrix is shown below. A has a probability p of showing 1 finger, probability q of showing 2 fingers, and probability 1 - p - q of showing 3 fingers. If B is equally likely to show 1, 2, or 3 fingers, what value of q maximizes the expected value of A’s payoff? You must prove your answer, not just make a guess.

			B	
		1	2	3
	1	+1	−1	+1
A	2	−1	+1	−1
	3	+1	−1	+1

9. Suppose that that there are two kinds of households, the Careless and the Careful; 99 percent of households are Careful and 1 percent are Careless. There is a 0.010 probability that fire will destroy a home inhabited by a Careless household, but only a 0.001 probability that fire will destroy a Careful home. If a home is destroyed by fire, what is the probability that it was occupied by a careless household?

10. Construct a model to analyze the effect of an influx of low-skill immigrant workers on wages. Keep your model as simple as possible, but your model must have the following features:
    a. There is one kind of output and two kinds of labor, low-skill and high skill.
    b. Every firm's output can be described by the same Cobb-Douglas production function with the elasticity of output with respect to low-skill labor lower than the elasticity of output with respect to high-skill labor.
    c. All firms are competitive and take wages and prices as given.
    d. Both labor supplies are perfectly inelastic.

Use your model to answer these questions (You must have an explicit mathematical model and you must use your model to formally derive your answers to these questions):

a. If there are equal amounts of both kinds of labor, are the wages of the low-skill workers higher than, lower than, or equal to the wages of high-skill workers?

b. If there is an increase in the supply of low-skill workers relative to high-skill workers, will the ratio of low-skill wages to high-skill wages increase, decrease, or stay the same?