Econ 58
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Midterm Answers

1. a. We can solve for K as a function of L and take the derivative holding Q constant:

$$K = \left(\frac{Q}{A}\right)^{1/\alpha} L^{-\beta/\alpha}$$

$$\frac{dK}{dL} = -\left(\frac{\beta}{\alpha}\right)\left(\frac{Q}{A}\right)^{1/\alpha} L^{-1-\beta/\alpha}$$

$$< 0$$

Alternatively, we can take the total derivative:

$$dQ = \frac{\partial Q}{\partial K}dK + \frac{\partial Q}{\partial L}dL$$

$$= \alpha\frac{Q}{K}dK + \beta\frac{Q}{L}dL$$

For dQ = 0,

$$\frac{dK}{dL} = -\frac{\beta}{\alpha}\frac{K}{L} < 0$$

b. The second derivative is

$$K = \left(\frac{Q}{A}\right)^{1/\alpha} L^{-\beta/\alpha}$$

$$\frac{d^2K}{dL^2} = \left(1+\beta/\alpha\right)\left(\frac{\beta}{\alpha}\right)\left(\frac{Q}{A}\right)^{1/\alpha} L^{-2-\beta/\alpha}$$

$$> 0$$

c. Substitute the equation for Q into the equation for the slope dK/dL:

$$\frac{dK}{dL} = -\left(\frac{\beta}{\alpha}\right)\left(\frac{AK^{\alpha}L^{\beta}}{A}\right)^{1/\alpha} L^{-1-\beta/\alpha}$$

$$= -\frac{\beta}{\alpha}KL^{\beta/\alpha}L^{-1-\beta/\alpha}$$

$$= -\frac{\beta}{\alpha}\frac{K}{L}$$

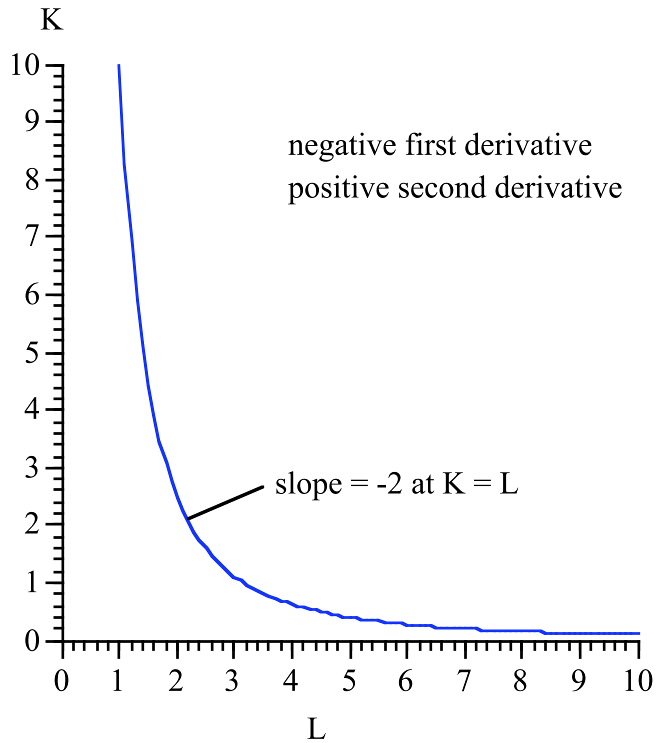
$$= -\frac{\beta}{\alpha} \text{ at } K = L$$

d. We immediately know from this equation in part (a)

$$K = \left(\frac{Q}{A}\right)^{1/\alpha} L^{-\beta/\alpha}$$

that the elasticity of K with respect to L is -□/□. This is consistent with the answer to part (c).

e. Here is a sketch



2.

a. I expect employment to typically be higher for (i) since an increase in l may increase profits while reducing profits per worker.

b. The firm's profits are

$$\begin{aligned} \pi &= PQ - WL - C \\ &= 5PL^{0.5} - WL - C \end{aligned}$$

The first derivative is equal to zero at

$$0 = \frac{\partial \pi}{\partial L} = 2.5PL^{-0.5} - W$$

$$L^{0.5} = \frac{2.5P}{W}$$

$$L = \frac{6.25P^2}{W^2} = \frac{6.25(20^2)}{10^2} = 25$$

c. Revenue per worker is

$$\begin{aligned} R &= \frac{PQ - C}{L} \\ &= 5PL^{-0.5} - CL^{-1} \end{aligned}$$

The first derivative is equal to zero at

$$0 = \frac{\partial R / L}{\partial L} = -2.5PL^{-1.5} + CL^{-2}$$

$$L^{0.5} = \frac{C}{2.5P}$$

$$L = \frac{C^2}{6.25P^2} = \frac{200^2}{6.25(20^2)} = 16$$

3. The profit function is

$$\pi = P_W Q_W + P_D Q_D - C$$
$$= 30Q_W + 300Q_D^{0.5} - 100 - 2\left(Q_D + Q_W\right)^{1.5}$$

The first-order conditions are

$$0 = \frac{\partial \pi}{\partial Q_W} = 30 - 3\left(Q_D + Q_W\right)^{0.5}$$

$$Q_D + Q_W = 10^2 = 100$$

$$0 = \frac{\partial \pi}{\partial Q_D} = 150Q_D^{-0.5} - 3\left(Q_D + Q_W\right)^{0.5}$$

$$150Q_D^{-0.5} = 30$$

$$Q_D = 5^2 = 25$$

Therefore:

a. $Q_W = 100 - 25 = 75$

b. $Q_D = 25$

c. $P_D = 300/\text{sqrt}(25) = 60$

d. yes:

$$MC = 3\left(Q_D + Q_W\right)^{0.5}$$
$$MR = P_W = 30$$

e. yes:

$$MC = 3\left(Q_D + Q_W\right)^{0.5}$$
$$MR = 150Q_D^{-0.5} = 150 / \sqrt{25} = 30$$

(NOT ASKED) The second-order conditions are

$$\frac{\partial^2 \pi}{\partial Q_W^2} = -1.5\left(Q_D + Q_W\right)^{-0.5} < 0$$

$$\frac{\partial \pi}{\partial Q_D^2} = -75Q_D^{-1.5} - 1.5\left(Q_D + Q_W\right)^{-0.5} < 0$$

$$\frac{\partial^2 \pi}{\partial Q_W \partial Q_D} = -1.5\left(Q_D + Q_W\right)^{-0.5}$$

with

$$\frac{\partial^2 \pi}{\partial Q_W^2}\frac{\partial \pi}{\partial Q_D^2} - \left[\frac{\partial^2 \pi}{\partial Q_W \partial Q_D}\right]^2 = \left[-1.5\left(Q_D + Q_W\right)^{-0.5}\right]\left[-75Q_D^{-1.5} - 1.5\left(Q_D + Q_W\right)^{-0.5}\right]$$

$$- \left[-1.5\left(Q_D + Q_W\right)^{-0.5}\right]^2$$

$$= \left[-1.5\left(Q_D + Q_W\right)^{-0.5}\right]\left[-75Q_D^{-1.5}\right]$$

$$> 0$$

4.

a. Taking the partial derivative with respect to R is

$$\frac{1}{M}\partial M = -0.25\frac{1}{R}\partial R$$

$$\frac{R}{M}\frac{\partial M}{\partial R} = -0.25$$

We can also find this elasticity by using exponentials to write the money demand equation as

$$M = e^{47}Y^{0.6}R^{-0.25}$$

which immediately shows that the interest elasticity is -0.25 (and the income elasticity is 0.60).

b. Similarly, the elasticity of money demand with respect to Y is 0.60:

$$\frac{1}{M}\partial M = 0.60\frac{1}{Y}\partial Y$$

If Y increases by 5%, M will increase by approximately 0.60(5%) = 3%.

c. Taking the total derivative:

$$dM = \frac{\partial M}{\partial Y}dY + \frac{\partial M}{\partial R}dR$$

$$\frac{dM}{M} = \left(\frac{\partial M}{\partial Y}\frac{Y}{M}\right)\frac{dY}{Y} + \left(\frac{\partial M}{\partial R}\frac{R}{M}\right)\frac{dR}{R}$$

$$= 0.60\frac{dY}{Y} - 0.25\frac{dR}{R}$$

For M and Y both to increase by 5%, R will have to fall by approximately 8%:

$$0.05 = 0.60(0.05) \square 0.25\frac{dR}{R}$$

$$\frac{dR}{R} = \frac{0.60(0.05) \square 0.05}{0.25}$$

$$= \square 0.08$$

5.

a. dA/A = (E – D)/A = (□A – □□A)/A = □(1 – □)

b. Because earnings are proportional to assets, they have the same growth rate, □(1 – □)

c. Because dividends are proportional to earnings, they have the same growth rate, g = □(1 – □)

d. Because price is proportional to dividends, they have the same growth rate, g = □(1 – □)

e. Because price and earnings grow at the same rate, their ratio is constant with a growth rate of 0.

f. Substitution gives this equation

$$P = \frac{D}{R \square g}$$

$$= \frac{\square\square A}{R \square \square(1 \square \square)}$$

g. The partial derivative is positive

$$\frac{\partial P}{\partial \square} = \frac{\left(R \square \square(1 \square \square)\right)\square A + \square\square A(1 \square \square)}{\left(R \square \square(1 \square \square)\right)^2}$$

$$= \frac{\square AR}{\left(R \square \square(1 \square \square)\right)^2}$$

$$> 0$$

h. An increase in the profit rate increases current dividends and also increases the growth rate of dividends, both of which increase the stock price.

i. The partial derivative is ambiguous:

$$\frac{\partial P}{\partial \square} = \frac{\left(R \square \square(1 \square \square)\right)\square A \square \square\square A\square}{\left(R \square \square(1 \square \square)\right)^2}$$

$$= \frac{\square A(R \square \square)}{\left(R \square \square(1 \square \square)\right)^2}$$

$$> 0 \text{ if } R > \square$$

$$< 0 \text{ if } R < \square$$

j. An increase in dividends at the expense of retained earnings only benefits shareholders if the rate of return on the assets purchased with these retained earnings is less than the shareholders' required rate of return.