Econ 58
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Fall 2006

Midterm Answers

1. a. Solving

$$y = a + b(y - c - fy)$$
$$= a - bc + b(1 - f)y$$
$$y = \frac{a - bc}{1 - b(1 - f)}$$

b. The expenditure multiplier m is

$$m = \frac{dy}{da} = \frac{1}{1 - b(1 - f)} > 0$$

c. The effect of an increase in f on this multiplier is

$$\frac{dm}{df} = -\frac{b}{\left(1 - b(1 - f)\right)^2} < 0$$

d. The key statistic is the marginal propensity to spend de/dy: by how much does spending increase when income increases by a dollar? The multiplier is 1 divided by 1 minus the marginal propensity to spend. With taxes, the marginal propensity to spend is equal to the marginal effect of income on disposable income times the marginal propensity to spend out of disposable income:

$$\frac{de}{dy} = \frac{de}{d(y - t)}\frac{d(y - t)}{dy}$$
$$= b(1 - f)$$

If the marginal tax rate is increased, then an increase in income has a smaller effect on disposable income and, as a consequence, spending, thus reducing the multiplier.

2. The first order condtion for revenue maximization is

$$0 = \frac{dR}{dt} = S + t\frac{dS}{dt}$$
$$t\frac{dS}{dt} = -S$$

a. Thus the elasticity of sales is -1:

$$\varepsilon = \frac{dS}{dt}\frac{t}{S} = -1$$

b. This makes sense. If the country increases the tax rate by 1% and sales fall by less than 1%, tax revenue increases; If the country increases the tax rate by 1% and sales fall by more than 1%, tax revenue falls. In the first case, the country can increase revenue by increasing the tax rate; in the second case, the country can increase tax revenue by reducing the tax rate. They are at the maximum only if sales fall by 1% when the tax rate increases by 1%.

3. The firm's revenue is

$$\begin{aligned} R &= P_1Q_1 + P_2Q_2 \\ &= (155 - Q_1)Q_1 + (205 - 2Q_2)Q_2 \\ &= 155Q_1 - Q_1^2 + 205Q_2 - 2Q_2^2 \end{aligned}$$

profits are

$$\begin{aligned} \pi &= R - C \\ &= 155Q_1 - Q_1^2 + 205Q_2 - 2Q_2^2 - 95 - 5Q_1 - 5Q_2 \\ &= 150Q_1 - Q_1^2 + 200Q_2 - 2Q_2^2 - 95 \end{aligned}$$

a. The first-order conditions are

$$0 = \frac{d\pi}{dQ_1} = 150 - 2Q_1$$

$$0 = \frac{d\pi}{dQ_2} = 200 - 4Q_2$$

The solutions are $Q_1 = 75$; $Q_2 = 50$.

b. The marginal revenue from producing one more unit of Q_1 is

$$\frac{dR}{dQ_1} = 155 - 2Q_1 = 155 - 2(75) = 5$$

c. The marginal revenue from producing one more unit of Q_2 is

$$\frac{dR}{dQ_2} = 205 - 4Q_2 = 205 - 4(50) = 5$$

d. We expect the answers to (b) and (c) to be the same because the firm will equate marginal revenue to marginal cost for each quantity, and marginal cost is 5 for both products.

4. Consider a firm whose assets and earnings at time t are described by these equations:

$$A_t = 32e^{0.05t}$$

$$E_t = 0.2A_t$$

a. The value of assets at time t = 0 is $A_0 = 32e^{0.05(0)} = 32$

b. The rate of growth of assets is 0.05 (5%)

c. Earnings are

$$\begin{aligned} E_t &= 0.2A_t \\ &= 0.2 * 32e^{0.05t} \\ &= 6.4e^{0.05t} \end{aligned}$$

The rate of growth of earnings is 0.05 (5%) too.

d. The rate of growth of the ratio of earnings to assets is 0: $\frac{E_t}{A_t} = \frac{0.2A_t}{A_t} = 0.2$

5. Using the model in the previous exercise,

a. Using a continuously compounded interest rate R, the present value V_t of earnings at time t is

$$
\begin{aligned}
V_t &= E_t e^{-Rt} \\
&= 6.4e^{0.05t} e^{-Rt} \\
&= 6.4e^{(0.05-R)t}
\end{aligned}
$$

b. The present value of earning increase or decrease as time passes depending on whether the interest rate is smaller or larger than 0.05 (5%):

$$V_t = 6.4e^{(0.05-R)t}$$

$$\frac{dV}{dt} = (0.05 - R)6.4e^{(0.05-R)t}$$

c. Because

$$D = \frac{D}{E}\frac{E}{A}A$$

the percentage change in dividends D is approximately equal to the percentage change in $\frac{D}{E}$ plus the percentage change in $\frac{E}{A}$ plus the percentage change in A.