

Midterm (75 minutes)

1. Consider this Cobb-Douglas production function:

$$Q = AK^\alpha L^\beta, \quad 0 < \alpha < 1, \quad 0 < \beta < 1$$

An isoquant is an equation that gives the combinations of K and L that can produce a fixed quantity Q.

a. Find the equation for the slope dK/dL of an isoquant for this production function. Is this slope positive or negative?

b. Find the equation for the curvature of an isoquant d^2K/dL^2 . Is this curvature positive or negative?

c. Find the equation for the slope of an isoquant at the point $K = L$.

d. Find the general equation for the elasticity of K with respect to L on an isoquant: $\epsilon = \frac{dK}{dL} \frac{L}{K}$

e. Sketch an isoquant for $\alpha = 1/3$, $\beta = 2/3$, with the units for both K and L ranging from 0 to 10.

2. Consider a firm whose output Q is related to its labor input L by this production function:

$$Q = 5L^{0.5}$$

The firm pays a constant wage W for each unit of labor and sells each unit of output for a constant price P ; it also has fixed costs C , no matter how much it produces. Consider these alternative organizational structures:

- i. The firm has an owner who chooses employment L that maximizes the firm's profits $\pi = PQ - WL - C$
 - ii. The firm is organized as a cooperative that chooses L that maximizes revenue per worker $R = (PQ - C)/L$.
- a. Explain why you expect the level of employment typically to be higher with (i) or (ii).

b. Determine the level of employment for (i) if $W = 10$, $P = 20$, and $C = 200$.

c. Determine the level of employment for (ii) if $W = 10$, $P = 20$, and $C = 200$.

3. Consider a firm that is a price-taker when it sells its product in world markets, but is able to charge whatever price it wants in its domestic market because the government has erected strong barriers to imports:

$$\text{cost: } C = 100 + 2(Q_D + Q_W)^{1.5}$$

$$\text{world price: } P_W = 30$$

$$\text{domestic price: } P_D = \frac{300}{\sqrt{Q_D}}$$

- a. Find the profit-maximizing level of this firm's world output Q_W .
- b. Find the profit-maximizing level of this firm's domestic output Q_D .
- c. Find the profit-maximizing level of this firm's domestic price P_D .
- d. Determine whether marginal revenue is equal to marginal cost for world output Q_W .
- e. Determine whether marginal revenue is equal to marginal cost for domestic output Q_D .
4. An economist estimated the following equation for the demand for money:
- $$\ln[M] = 47 + 0.6 \ln[Y] - 0.25 \ln[R]$$
- where M is the quantity of money demanded, Y is income, and R is the interest rate.
- a. What is the value of the estimated interest elasticity of money demand, $\square = \frac{\partial M}{\partial R} \frac{R}{M}$
- b. If R is constant and Y grows by 5%/year, approximately how fast will money demand grow?
- c. If Y increases by 5%, by how much must R increase or decrease for M to increase by 5%?

5. Consider this constant dividend-growth model of a firm's stock price P , dividends D , earnings E , and assets A , where g is the growth rate of dividends and R is the shareholders' required rate of return:
- stock price: $P = D/(R - g)$
 dividends: $D = \beta E$
 earnings: $E = \alpha A$
 change in assets: $dA = E - D$
- What is the growth rate of assets dA/A ?
 - What is the growth rate of earnings, dE/E ?
 - What is the growth rate of dividends, $g = dD/D$?
 - What is the growth rate of the price, dP/P ?
 - What is the growth rate of the price-earning ratio, $d(P/E)/(P/E)$?
 - Derive an equation that demonstrates that P is a function of only A , β , R , and α .
 - Use the equation in (f) to find the partial derivative of P with respect to β ; is it positive or negative?
 - Explain the economic logic of your answer to (g).
 - Use the equation in (f) to find the partial derivative of P with respect to α ; is it positive or negative?
 - Explain the economic logic of your answer to (i).