Econ 58
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Midterm (75 minutes)

Answer all 5 questions. **Always** show your work. No Calculators allowed.

1. Suppose that a person's utility U depends on two items, X and Y, in this way

$$U[X,Y] = A\sqrt{XY}$$

where A is a positive parameter.

a. Holding Y constant, is the first derivative of utility with respect to X positive, zero, or negative?

b. Holding Y constant, is the second derivative of utility with respect to X positive, zero, or negative?

c. An indifference curve with X on the horizontal axis and Y on the vertical axis shows those combinations of X and Y that give a constant level of utility. Set utility equal to a constant amount U_0 and find the equation for the indifference curve that gives Y as a function of X.

d. What is the slope, dY/dX, of the indifference curve at the point X = Y = 10?

e. Is the second derivative d^2Y/dX^2 of the indifference curve positive, zero, or negative?

f. Sketch the indifference curve.

g. Interpret the value of dY/dX determined in Part (d); for example, if $dY/dX = 5$, does this mean that Y is 5 times X, that the utility from Y is 5 times the utility from X, or what?

h. If this person is at the point $X = Y = 10$ and the price of X is \$1 and the price of Y is \$2, will this person want to buy more X and less Y, buy more Y and less X, or stay at $X = Y = 10$?

2. Suppose that a firm's output is characterized by this production function

$$Q = AK^{□}L^{□}$$

where Q is the amount produced; K and L are the amounts of capital and labor employed; and A, □, and □ are positive parameters. The cost of employing capital and labor is given by this equation

$$C = rK + wL$$

where r is capital's rental rate and w is labor's real wage rate.

a. Interpret the parameter □.

b. Use the Lagrangian method to determine the equation for the capital-labor ratio K/L that minimizes the cost C of producing output Q = 100.

c. When the firm is minimizing the cost of producing a given output (as in Part (b)), how is the ratio of the marginal products

$$\frac{\partial Q / \partial K}{\partial Q / \partial L}$$

related to the rental-wage ratio r/w?

d. Now suppose that $\frac{\partial Q}{\partial K} = 1$, $\frac{\partial Q}{\partial L} = 3$, $r = 0.1$, and $w = 0.2$. Should the firm employ more capital and less labor, more labor and less capital, or stick with the current capital and labor?

e. Explain you answer to Part (d) in ordinary English, using no mathematical symbols.

3. Suppose that a monopolist produces the same product in two different locations

$$C_1 = 20 + Q_1^2$$

$$C_2 = 30 + Q_2^2$$

where C_1 and C_2 are the respective costs of producing quantities Q_1 and Q_2. Demand is described by this equation relating the market price to total output:

$$P = 100 - 2(Q_1 + Q_2).$$

a. Find the quantities Q_1 and Q_2 that maximize the firm's profit

$$\pi = P(Q_1 + Q_2) - (C_1 + C_2)$$

b. Explain the economic logic for why the profit-maximizing value of Q_1 is larger than, smaller than, or equal to Q_2, using words not symbols.

4. Consider this model of government fiscal policy

$$g = g_0 e^{\square t}$$

$$h = \square y$$

$$z = g \,\square\, h$$

$$y = y_0 e^{\square t}$$

where g is government spending, h is taxes, z is the government deficit, y is national income, and t is time. The endogenous variables g, h, z, and y are all in real terms; g_0 and y_0 are exogenous variables and □, □, and □ are constant parameters.

a. Interpret the exogenous variable g_0.

b. Interpret the parameter □.

c. Find the equation for determining z/y, the ratio of the deficit to national income.

d. If z is currently positive, determine the conditions under which z/y falls over time.

e. State the conclusion in part (d) in plain English, using no mathematical symbols.

5. Suppose that a fraction $\square_1$ of wealth is invested in a risky asset whose return has a mean $\square_1$ and standard deviation $\square_1$, a fraction $\square_2$ is invested in a risky asset whose return has a mean $\square_2$ and standard deviation $\square_2$, and that the returns on these two risky assets are perfectly negatively correlated. The variance of the portfolio return is

$$\square^2 = \square_1^2\square_1^2 + \square_2^2\square_2^2 \,\square\, 2\square_1\square_2\square_1\square_2 \quad , \quad 0 < \square_1 < 1,\ 0 < \square_2 < 1,\ \square_1 + \square_2 = 1$$

Now suppose that $\square_1 = \square_2$.

a. Use the Lagrangian method to determine the value of $\square_1$ that minimizes $\square^2$.

b. What is the minimum value of $\square^2$?

c. How do you know that the solution in Part (a) is a minimum?.