Complex IRRs Do Not Resolve the NPV-IRR "Debate"

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Abstract

A net present value equation with *n* time periods has *n* internal rates of return, some of which may be complex numbers. Despite claims to the contrary, these complex numbers are not economically meaningful and do not alter the fundamental fact that an IRR criterion is flawed.

key words: net present value, internal rate of return

running head: The NPV-IRR Debate

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The net present value (NPV) and internal rate of return (IRR) are popular alternative criteria for assessing investment projects. A heavily downloaded and widely cited paper with the hopeful title, "A Resolution to the NPV–IRR Debate?" (Osborne 2010) argues that these two approaches can be reconciled by considering complex IRRs. However, complex IRRs are not economically meaningful and do not resolve any debate. Indeed, there is no debate to be resolved.

The NPV and IRR

The logic behind an NPV calculation is straightforward. The net present value of a cash flow is the sum of the present values of the components X_t of the cash flow, each discounted by the investor's required return R_t:

$$NPV_t = \sum_{t=0}^{n} \frac{X_t}{(1+R_t)^t} \tag{1}$$

The NPV criterion is that an investment is worth undertaking if the NPV is positive.

The internal rate of return is the single discount rate I that makes the NPV equal to zero:

$$0 = \sum_{t=0}^{n} \frac{X_t}{(1+I)^t} \tag{2}$$

The IRR criterion is that an investment is financially attractive if its IRR is larger than the investor's required return (the *hurdle rate*).

Many business managers prefer the IRR (Block 1997, Burns & Walker 1997, Graham & Harvey 2001), apparently because they interpret the IRR as the project's rate of return. The IRR's appeal is also enhanced by the fact that it is mathematically identical to a bond's yield to maturity, which is widely interpreted as a bond's rate of return. The NPV, in contrast, has no straightforward interpretation as a rate of return, particularly if there are outlays after the initial period. Nonetheless, there are several insurmountable problems with an IRR criterion.

IRR Problems

Figure 1 shows the NPV for different required returns for an investment that costs $100 and yields $10 a year forever. The IRR is where the NPV line crosses the horizontal axis. The IRR is the break-even required return in that the NPV is positive if the required return is less than the IRR and negative otherwise. Here, the NPV and IRR criteria agree.

There are other cash-flow scenarios in which this is not the case, most fundamentally because the IRR does not tell us whether the NPV is positive or negative for required returns other than the IRR. If the IRR is, say, 20%, this does not tell us whether the NPV is positive or negative for a required return of 10%. This inadequacy underlies a plethora of pitfalls.

Insensitivity to sign changes in the cash flow. If the signs of the cash flow X_i are reversed, Equations 1 and 2 show that the IRR does not change, but the sign of the NPV is reversed for all required returns other than the IRR. Figure 1 shows the NPV and IRR for an initial investment followed by positive cash flows for the life of the project. If, instead, a football team replaces dozens of seats in its football stadium with luxury boxes, there is an initial profit from the sale of the luxury boxes but lost ticket revenue forever from the removed seats. The NPV curve is flipped vertically so that the IRR is unchanged but now the NPV is *positive* if the required return is larger than the IRR because the financial burden of the lost future income is smaller, the larger is the required return.

Multiple IRRs. Many investments are more complicated that a payment followed by income, or a receipt followed by expenses. There may be an initial expense, then income, then expenses. A water company, for example, incurs costs building an infrastructure that generates years of profits, but also incurs large costs at some point when it replaces and/or updates its infrastructure. We know from Descartes' sign rule that there may be multiple IRRs when there is more than one sign change in the cash flow. Although the multiple IRRs give us the break-even required returns where the NPV is zero, they do not tell us whether the NPV is positive or negative for required returns above, below, or in-between the IRRs.

No IRRs. For some cash flows, there is no real-valued IRR because the NPV is positive (or negative) for all required returns.

A Term Structure of Required Returns. If the term structure of interest rates on Treasury zeros is not flat, the yields to maturity on coupon-paying Treasuries should not be compared to each other or to a single required return. If, for example, the term structure is upward sloping, the yields to maturity on correctly priced Treasury bonds are lower for high-coupon bonds than for low-coupon bonds. This is why bonds are priced using the term structure, not the yield to maturity. The same is true of other investments. If the term structure of required returns is not flat (R_t is not constant in Equation 1), the IRR criterion is useless because IRRs should not be compared to each other or to a single required return.

Mutually Exclusive Projects. Often, a firm must choose between mutually exclusive projects. Should the height of a building be 10 stories or 20 stories? Should the heating system be electric or gas? Whenever the NPV curves for mutually exclusive projects cross, a comparison of the IRRs can be misleading.

IRRs Are Not Additive. If Project A has a higher IRR than Project B, the combination of Project A and Project C may have a lower IRR than the combination of Projects B and C. NPVs do not have this undesirable feature. If A has a higher NPV than B, then A and C has a higher

NPV than B and C.

A Resolution to the NPV–IRR Debate?

Most IRR pitfalls are well known (for example, Magni, 2013). Osborne proposes "resolving the debate" by noting that in a net present value equation with n time periods, there are n internal rates of return, some of which may be complex numbers. He also derives an equation showing how the NPV is mathematically related to these n IRRs. However, the existence of complex IRRs does not resolve *any* of the issues enumerated above, and introduces the additional issue that complex IRRs are not economically meaningful. If the most important weakness of the NPV criterion is that it does not yield a rate of return that businesspeople can understand easily, this weakness is not solved by presenting businesspeople with complex-valued IRRs.

Osborne argues that the existence of multiple real and complex IRRs is not a problem because the multiple IRRs can be used to determine the NPV. True enough. However, Osborne doesn't recommend using the IRRs for anything other than determining the NPV, and we don't need the IRRs in order to calculate the NPV. The fact that NPV is mathematically related to the n IRRs does not challenge the superiority of the NPV criterion or provide a rationale for looking at these n IRRs.

Osborne uses this example to demonstrate the usefulness of complex IRRs. If you invest \$1 now and spend another \$1 two years later with nothing to show for it, there are no real-valued IRRs but there are two complex-valued IRRs: $-1 + i$ and $-1 - i$. However, not only are these complex IRRs useless for comparison to the investor's required return, there is still the essential problem that IRRs, real or complex, are insensitive to sign changes in the cash flow.

Suppose, in contrast to Osborne's example, that you are fortunate enough to receive \$1 now and \$1 two years later, perhaps by selling financial derivatives. The IRRs are $-1 + i$ and $-1 - i$, the same as in Osborne's example of investing \$1 now and \$1 two years from now. These IRRs are of no use in choosing between these two investments because the IRRs are exactly the same. In contrast, an NPV calculation correctly shows that the second investment is preferred to the first investment because it has a positive NPV for any required return while the first investment has a negative NPV for any required return.

Conclusion

Real IRRs are break-even required returns where the NPV is zero, but they do not tell us whether the NPV is positive or negative for other required returns. Complex IRRs do not alter this fundamental fact and, in addition, are not economically meaningful.

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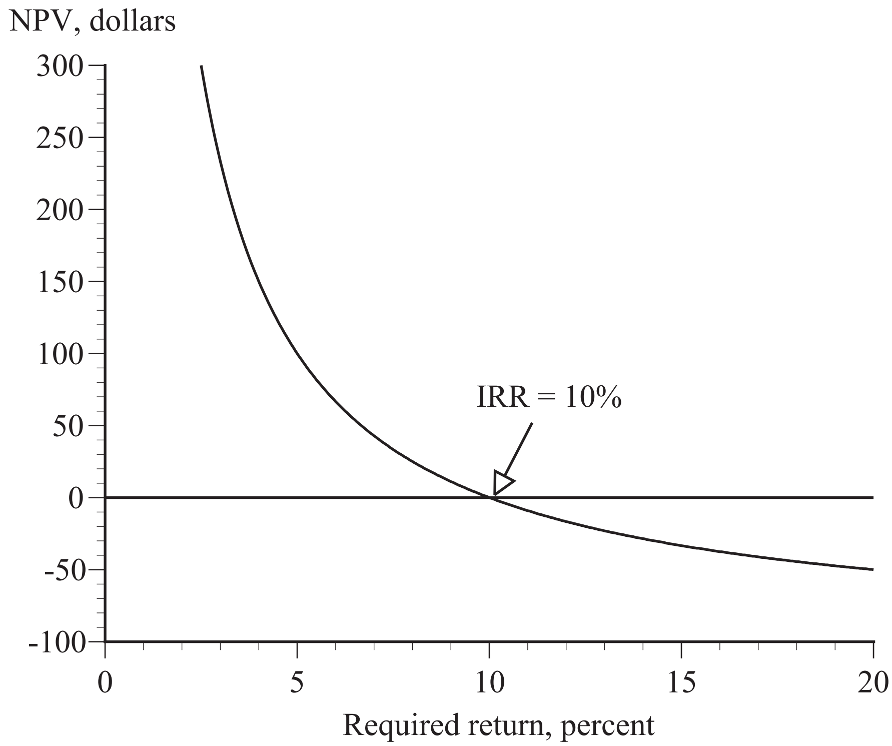

Figure 1 The IRR is a Break-Even Required Return