

Stocks Should Be Valued with a Term Structure of Required Returns

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The most fundamental question in investing is *valuation*, the amount an investor is willing to pay for a prospective cash flow. For bonds, the coupons and maturation value are routinely discounted by a term structure of required returns. For stocks, in contrast, dividends or other measures of the cash flow are generally discounted by a single required return, typically the interest rate on a Treasury bond of a prespecified maturity plus a risk premium.

It is well known that, except in the special (and rare) case of a flat term structure, it is a mistake to value bonds using a single interest rate. It is not so well known that the same is true of stocks. One exception is Smith [1991]. Another is John Burr Williams [1938, p. 359], who wrote that “A *constant* rate is wrong, and can only lead to wrong results.”

Brennan [1997] and Ang and Liu [2004] criticized the capital asset pricing model (CAPM) for using a single risk-free rate and constant market risk premium. They developed models for estimating time-varying market discount rates that are consistent with CAPM and observed stock market prices. We look at the problem the other way around, from the perspective of investors trying to determine whether stocks are cheap or expensive. Instead of using stock market prices to generate discount rates, we use discount rates to value stocks. We argue that investors should use time-varying discount

rates (presumably based on the term structure of Treasury interest rates), and we illustrate the valuation errors that can occur if a single, constant discount rate is used instead.

VALUING BONDS

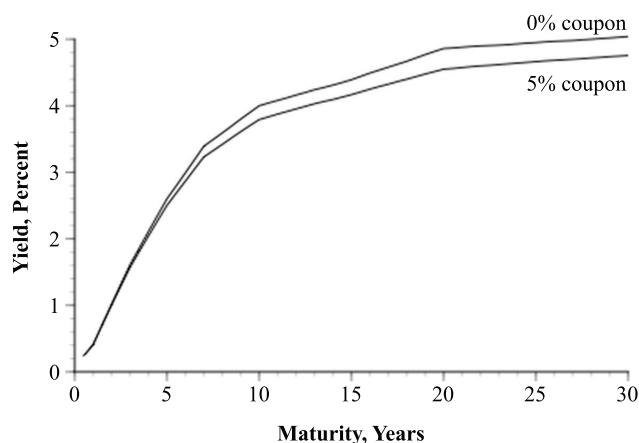
Investors regularly value bonds by discounting the coupons and maturation value by the appropriate required return for each payment date; for example, a coupon six months from now should be discounted by the required return on a six-month zero. Thus, the present value of a bond’s coupons C and maturation value M after n periods, using a term structure of required returns r_i on zeros, is

$$P = \frac{C}{(1+r_1)^1} + \frac{C}{(1+r_2)^2} + \dots + \frac{C}{(1+r_n)^n} + \frac{M}{(1+r_n)^n} \quad (1)$$

A bond’s yield to maturity is the constant interest rate y that gives the same present value as does the complete term structure:

$$\frac{C}{(1+y)^1} + \frac{C}{(1+y)^2} + \dots + \frac{C}{(1+y)^n} + \frac{M}{(1+y)^n} = \frac{C}{(1+r_1)^1} + \frac{C}{(1+r_2)^2} + \dots + \frac{C}{(1+r_n)^n} + \frac{M}{(1+r_n)^n} \quad (2)$$

EXHIBIT 1 Yield Curves, March 31, 2010



Unless the term structure is flat, a bond's yield depends on its coupon rate. Specifically, if the term structure is upward sloping, the yield to maturity on an n -period coupon bond will be somewhat less than the interest rate on an n -period zero, and the reverse is true if the term structure is downward sloping. This principle is illustrated in Exhibit 1 using the Treasury term structure on March 31, 2010 (Office of Thrift Supervision [2010]) to determine the yields on zeros and 5% coupons bonds for maturities ranging from six months to 30 years. More generally, correctly valued Treasury bonds with the same maturity (e.g., 20-year 3% and 5% coupon bonds) usually have different yields, making the yields a meaningless measure of their attractiveness.

Looked at from a valuation perspective, when the term structure is not flat, there are two pitfalls in using a single interest rate to value bonds, one obvious and one subtle. The obvious mistake is to use a single interest rate to value bonds with different maturities (e.g., using the three-month T-bill rate to value 20-year bonds). Using a single interest rate to value bonds with different maturities will overvalue long-term bonds relative to short-term bonds when the term structure is upward sloping and undervalue them when the term structure is downward sloping.

The subtle error is to use a single interest rate to value bonds with the same maturity but different coupon rates (e.g., using the same interest rate to value 3% and

5% coupon 20-year bonds). Using a single interest rate to value bonds with different coupon rates will undervalue high-coupon bonds relative to low-coupon bonds when the term structure is upward sloping and overvalue them when the term structure is downward sloping. To avoid these pitfalls, bonds should be valued using the full term structure, and investors generally do so.

Warren Buffett has famously argued that investors should think of a stock as a "disguised bond" (Loomis [2001]), so it is not surprising that using a single required return to value stocks is just as wrong as using a single interest rate to value bonds. Yet, unlike bonds, investors generally use a single discount rate.

VALUING STOCKS

Decades ago, investing was haphazard. Investors figured that a stock was worth what people were willing to pay, and the game was to guess what people would pay tomorrow for stocks bought today. Then John Burr Williams [1938] wrote his classic treatise, *The Theory of Investment Value*, which unleashed a revolution by arguing that investors could use the present value of dividends to estimate a stock's intrinsic value, which could then be compared to market prices. The very first sentence of Williams's book is "Separate and distinct things not to be confused, as every thoughtful investor knows, are real worth and market price." A stock is worth buying if its intrinsic value is higher than its price and not otherwise. This idea is the cornerstone of value investing.

Williams's reasoning is the basis for Benjamin Graham's [1959] imaginary Mr. Market, who comes by every day offering to buy the stock you own or to sell you more shares. Sometimes Mr. Market's price is reasonable. Other times, it is silly. There is no reason for your assessment of a stock's value to be swayed by Mr. Market's prices, although you may sometimes take advantage of his foolishness. This is also the basis for Warren Buffett's variation on this theme (Wiley [2017]): "I never attempt to make money on the stock market. I buy on the assumption that they could close the market the next day and not reopen it for five years."

An intrinsic value calculation is based on an investor's required return (what Williams called a *personal rate of interest*), which depends on the returns available on Treasury bonds and other investments.

Williams deduced that a stock's intrinsic value P should be determined by discounting dividends D_i by a term structure of required returns R_i :

$$P = \frac{D_1}{(1+R_1)} + \frac{D_2}{(1+R_2)^2} + \frac{D_3}{(1+R_3)^3} + \dots \quad (3)$$

A dividend three months from now should be discounted by a three-month required return and a dividend 10 years hence by a 10-year required return.

The Yield to Perpetuity

As a simplification, instead of using the term structure of Treasury rates to construct a term structure of stock required returns, Williams used the *yield to perpetuity* on Treasury bonds: the constant interest rate π that gives the same present value for an unending constant income stream as does the actual term structure of Treasury rates r_i :

$$\begin{aligned} \frac{1}{(1+\pi)} + \frac{1}{(1+\pi)^2} + \frac{1}{(1+\pi)^3} + \dots \\ = \frac{1}{(1+r_1)} + \frac{1}{(1+r_2)^2} + \frac{1}{(1+r_3)^3} + \dots \end{aligned} \quad (4)$$

For extrapolating Treasury rates beyond the horizon n of the longest Treasury bond being traded, Williams assumed that their forward rates are equal to the last observed forward rate:

$$F_{n-1,n} = \frac{(1+r_n)^n}{(1+r_{n-1})^{n-1}} - 1$$

Therefore, the assumed interest rates beyond the last observed Treasury rate are given by

$$(1+r_k)^k = (1+r_n)^n (1+F_{n-1,n})^{k-n} \quad \text{for } k > n$$

Using this relationship and simplifying Equation (4), we have

$$\begin{aligned} \frac{1}{\pi} = \frac{1}{(1+r_1)} + \frac{1}{(1+r_2)^2} + \dots \\ + \frac{1}{(1+r_n)^n} + \frac{1}{(1+r_n)^n} \frac{1}{F_{n-1,n}} \end{aligned} \quad (5)$$

Williams used a required stock return equal to the yield to perpetuity on Treasury bonds plus a risk premium

of a few percentage points. In the case studies in his 1938 treatise, he used a 3% yield to perpetuity on Treasury bonds and required returns of 5.25% for General Motors and U.S. Steel and 5% for Phoenix Insurance.

The Constant-Growth Model

Williams' yield to perpetuity never caught on. Instead, investors today generally use the yield to maturity on a single Treasury bond of a prespecified maturity to determine a stock's constant required return R , often in conjunction with the constant-growth model, which assumes that dividends grow forever at a steady rate g . A stock's intrinsic value, P , is then given by this familiar equation, also derived by Williams:

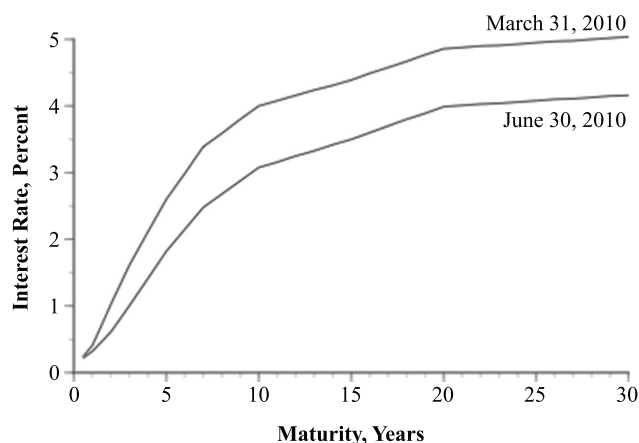
$$P = \frac{D_1}{R-g} \quad (6)$$

Earlier, we noted two reasons why it is a mistake use a single interest rate to value bonds. These are also reasons why it is a mistake to use a single interest rate to value stocks. First, just as a 20-year bond should not be valued using the three-month T-bill rate, a stock's dividends over an indefinite horizon should not be valued using the three-month T-bill rate. Second, just as the same interest rate should not be used to value 3% and 5% coupon 20-year bonds, the same required return should not be used to value stocks with 2% and 4% growth rates. (The use of a single interest rate will overvalue low-coupon bonds and growth stocks when the term structure is upward sloping and undervalue them when the term structure is downward sloping.)

Nonetheless, instead of using the entire term structure, academics and practitioners typically use a single interest rate on bonds with a fixed maturity, plus a constant risk premium, for the required return. Short-term rates are the most popular. For example, Copeland and Westin [2004]; Body, Kane, and Marcus [2010]; Body and Merton [2000]; and Yardeni [1994] use the three-month Treasury-bill rate. Brealey, Meyers, and Allen [2014]; Brown [1988]; and Ross, Westerfield, and Jaffe [2006] use one-year Treasury bills. Einhorn and Shangquan [1984], Bank of America Merrill Lynch [2014], and Goldman Sachs [2013] use five-year Treasury bonds. Yardeni [1997] and JPMorgan [2006] use the 10-year Treasury yield. Bing and Wilcox [1983] use the yield on 30-year Treasury bonds.

EXHIBIT 2

Treasury Zero Rates, March and June of 2010



In CAPM models, required returns almost invariably use Treasury bills as the risk-free asset. The use of a short-term interest rate is sometimes rationalized by the presumption that investors intend to hold stocks for only a few months. This is akin to saying that investors who intend to sell a 30-year Treasury bond after three months should value the bond by discounting 30 years of coupons by the three-month T-bill rate. This argument is clearly wrong for bonds, and it is also wrong for stocks. The intrinsic value of a bond or stock is the present value of the cash generated by the asset and does not depend on the holding period because an intrinsic-value calculation assumes that the asset will never be sold (Williams [1938]).

At the other end of the term structure, Brigham and Earhardt [2010] observed that short-term and long-term bond rates do not move in locked step and then chose a long-term Treasury rate because it is less volatile than short rates. If low volatility were a compelling criterion, we might as well use 0%, the never-changing return on cash.

Goldman Sachs [1988] used an even more bizarre criterion when it advised clients that, if the inflation rate were used in place of interest rates in the dividend-discount model, the S&P 500 would appear to be fairly valued rather than 15% overvalued. That works if we are more concerned with the conclusion than the assumptions. An investor's required return surely depends on the returns available on comparable investments, such as Treasury bonds. If the rate of inflation is zero and the

one-year Treasury rate is 5%, intelligent investors would not pay a dollar today to get a dollar a year from now because a dollar today can earn 5%.

The insurmountable problem with using a single interest rate—short or long—is that the resultant valuations zig and zag unreasonably as the term structure twists and turns. Exhibit 2 shows the term structure for Treasury zeros on March 31, 2010 and June 30, 2010. The six-month and one-year rates dipped slightly (by two and nine basis points, respectively), whereas longer-term rates fell by nearly a full percentage point. Investors using short-term rates would hardly change their valuation estimates. If, however, the duration of the S&P 500 is about 25 (Blitzer, Luo, and Soe [2012]), investors using long-term rates would reduce their valuations drastically. Did intrinsic values dip slightly or drop dramatically? Only a full term structure can answer that question.

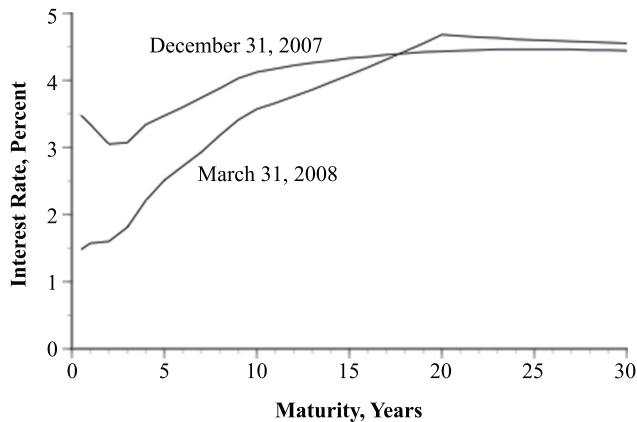
Exhibit 3 shows another example. Interest rates beyond 18 years increased between December 31, 2007, and March 31, 2008, whereas shorter-term rates collapsed. Investors using short-term rates and those using long-term rates would have very different stock valuations during these months. Specifically, investors using long-term rates would conclude that stock valuations had fallen, and investors using short-term rates would draw the opposite conclusion. They could both do better using the complete term structure.

Does It Make a Difference?

Damodaran [2008] argued that, as a practical matter, it makes little difference if a single required return is used in place of a term structure of required returns: “[W]ith any well reasonably well behaved yield curve, the effect on present value of using year-specific risk-free rates is likely to be small, since the rates do not deviate significantly across time.” Perhaps the examples in Exhibits 2 and 3 are extreme and rare, and the term structure is generally well behaved. Perhaps the use of a single interest rate in place of a complete term structure is a useful shortcut that gives intrinsic values close to those implied by the full term structure.

To investigate this question, we used the Shiller–McCulloch [1989] term structure data. They report that these “are the cleanest interest rate data available, in that they are based on a broad spectrum of government bond prices, and are corrected for coupon

EXHIBIT 3 Treasury Zero Rates, December 2007 and March 2008



and special tax effects.” These data cover a 40-year period, December 1946 through February 1987, and have the added advantage that we now have several decades of subsequent dividend data. Smith [1991] estimated the valuation errors caused by a single required return using a variety of assumed dividend growth rates. Instead of relying on hypothetical assumptions about future dividends, as Smith did, we will use actual dividends to compare valuation models. We do not know how close these actual dividends were to the projections used by investors because these projections are unknown. We do know how large the valuation errors would have been if investors had forecast dividends accurately but had used a single interest rate in place of the complete term structure.

In the same spirit, we do not know the market’s required return that was used to discount dividends. Indeed, there is no single market return. Some investors use short-term rates; others use long-term rates. Some do not use interest rates at all, relying instead on price-earnings ratios, price charts, and various measures of investor sentiment. Throw in what Keynes called “animal spirits,” and it is unrealistic to think that Mr. Market is using one interest rate to determine market prices.

More fundamentally, our research question concerns investors who are trying to estimate intrinsic values, which can then be compared to Mr. Market’s prices. Does it make much difference if value-investors use a single required return as a shortcut in place of a complete term structure of required returns?

Specifically, how close are valuations based on a single interest rate to valuations using the full term structure? We label the differences *valuation errors*.

The Shiller–McCulloch data are estimates of the continuously compounded interest rates on Treasury zeros with maturities ranging from one month to 25 years. The Appendix explains how we interpolated and extrapolated their data. We used monthly S&P 500 dividend data for the period January 1947 through December 2015. After December 2015, we assumed an instantaneous dividend growth rate of 5.5%, the trend growth rate for the earlier years.

These interest rate and dividend data were used to value stocks using six discount models:

1. term structure of Treasury zeros
2. three-month Treasury bills
3. one-year Treasury bills
4. 10-year coupon bonds
5. 25-year Treasury zeros
6. yield to perpetuity

Our baseline intrinsic-value calculations use the complete term structure with a 4% risk premium, the value used by Swensen [2000] for comparing the S&P 500 and long-term Treasury bonds, but our results are robust to other values. During this time period, long-term rates were, on average, higher than short-term rates (the average spread between 25-year Treasury bonds and three-month Treasury bills was 0.99%). If we also used a 4% risk premium for the single rate models, there would be a built-in bias in the valuations in that intrinsic-value calculations based on short-term rates would, on average, be higher than those based on long-term rates. We consequently set the risk premium for each of the five single-rate models so that the average intrinsic value is the same for all models during the time period we analyzed and is equal to the average intrinsic value for the baseline model using the complete term structure. The risk premiums turned out to be approximately 4% for the single-rate models using long-term interest rates and around 5% for the single-rate models using short-term interest rates.

Exhibit 4 shows the large magnitude and volatility of the estimation errors when the three-month T-bill rate is used as the risk-free rate. The results are similar when the one-year T-bill rate is used as the risk-free rate.

EXHIBIT 4

Estimation Errors, Three-Month Treasury Bills

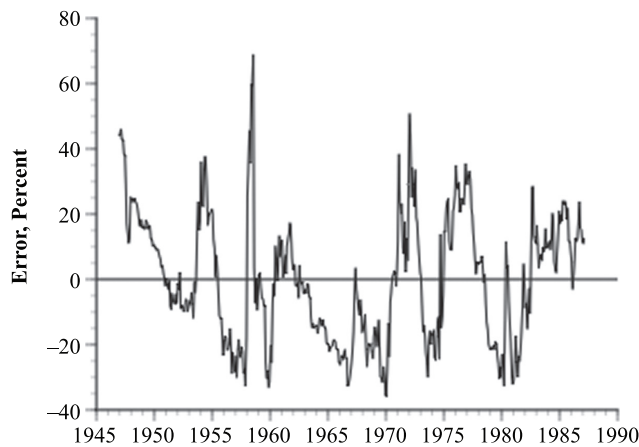


EXHIBIT 5

Mean Absolute Errors (MAE) and Root Mean Square Errors (RMSE)

Risk-Free Rate	Risk Premium	MAE	RMSE
Three-month T-bills	5.20	16.20	19.39
One-year T-bill	4.86	15.56	19.13
10-year T-bond	4.04	5.38	6.99
25-year zero	4.02	1.89	2.57
Yield to Perpetuity	4.00	0.26	0.35

Exhibit 5 shows the mean absolute errors and root mean square errors for the five constant-discount-rate models. The two short-term rates are much more inaccurate than the longer-term rates. Williams' yield to perpetuity is clearly the closest proxy to the full term structure.

Exhibit 6 makes this conclusion more concrete by tabulating how often, out of 483 valuations, the valuations based on a constant required return differed by more than 5%, 10%, or 20% from the valuations using the full term structure. For the three-month and one-year Treasury-bill rates, more than 60% of the errors were larger than 10%, and more than 30% of the estimates were off by more than 20%. For 10-year bonds, 40% of the errors were larger than 5%, and 15% were greater than 10%. The longer maturities were more accurate proxies for the complete term structure, with the yield to perpetuity being the most accurate of all.

EXHIBIT 6

The Number of Large Errors Out of 483 Valuations

Risk-Free Rate	Risk Premium	Estimates Were Off by More Than		
		5%	10%	20%
Three-month T-bills	5.20	407	324	174
One-year T-bill	4.86	377	310	158
10-year T-bond	4.04	195	74	2
25-year zero	4.02	32	1	0
Yield to Perpetuity	4.00	0	0	0

The errors using the three-month and one-year T-bill rates were not due to the fact that the term structure, on average, happened to be upward sloping during the period studied. We adjusted the risk premium to account for that. Instead, the large errors reflect the fact that short-term rates are much more volatile than the full term structure.

There would be even more dispersion among the valuations if we calculated the intrinsic values of individual stocks with widely varying dividend projections.

CONCLUSION

Stocks, like bonds, should be valued using a complete term structure of required returns. However, investors generally use one interest rate (e.g., the three-month T-bill rate) plus a constant risk premium. A single interest rate is a noisy proxy for a complete term structure and causes a variety of valuation errors, including a relative overvaluation of growth stocks when the term structure is upward sloping and an undervaluation when the term structure is downward sloping. More generally, twists and turns in the term structure cause valuations based on a single interest rate to fluctuate substantially around valuations based on the complete term structure. If anything, these valuation errors have most likely become even larger in recent years as short-term rates have dropped near zero, creating a very steep term structure.

For those investors who insist on using a single interest rate, the interest rate that provides the best approximation of a complete term structure is the yield to perpetuity, recommended by John Burr Williams—which, as far as we know, no one uses. The worst proxies are short-term rates—which are also the most popular.

APPENDIX

We interpolated interest rates for missing maturities using the forward rates implied by the requirement that a dollar invested for t_1 years at a continuously compounded rate r_1 and then reinvested between years t_1 and t_2 at the forward rate $f_{1,2}$ gives the same final wealth as a dollar invested for t_2 years at r_2 :

$$e^{r_1 t_1} e^{f_{1,2}(t_2-t_1)} = e^{r_2 t_2}$$

$$f_{1,2} = \frac{r_2 t_2 - r_1 t_1}{t_2 - t_1} \quad (\text{A-1})$$

The implied continuously compounded interest rate r_t for a zero maturing at time t during the interval between t_1 and t_2 is given by this equation:

$$e^{r_t t} = e^{r_1 t_1} e^{f_{1,2}(t-t_1)}$$

$$r_t = \frac{r_1 t_1 + f_{1,2}(t-t_1)}{t} \quad (\text{A-2})$$

If t_2 is the last observed interest rate, we can extrapolate interest rates beyond this point by assuming that the last forward rate is constant, the same assumption made by Williams in calculating the yield to perpetuity. The implied instantaneous interest rate r_t for a zero maturing at time $t > t_2$ is given by

$$e^{r_t t} = e^{r_2 t_2} e^{f_{1,2}(t-t_2)}$$

$$r_t = \frac{r_2 t_2 + f_{1,2}(t-t_2)}{t} \quad (\text{A-3})$$

The yield to perpetuity is given by this relationship:

$$e^{-\gamma^1} + e^{-\gamma^2} + e^{-\gamma^3} + \dots = e^{-r_1^1} + e^{-r_2^2} + e^{-r_3^3} + \dots \quad (\text{A-4})$$

The last observed one-period forward rate r_1^{n-1} is determined by

$$e^{r_n^n} = e^{r_{n-1}^{(n-1)}} e^{f_{n-1,n}}$$

The implied interest rates r_k beyond the last observed rate r_n are given by this equation ($k > n$):

$$e^{r_k^k} = e^{r_n^n} e^{f_{n-1,n}(k-n)}$$

Simplifying Equation (A-4), we have

$$\frac{1}{e^\gamma - 1} = e^{-r_1^1} + e^{-r_2^2} + e^{-r_3^3} + \dots + e^{-r_n^n} + e^{-r_n^n} \frac{1}{e^{f_{n-1,n}} - 1} \quad (\text{A-5})$$

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