The Real Dogs of the Dow

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#### Abstract

Kahneman and Tversky argue that regression to the mean is a pervasive but subtle statistical principle that is often misunderstood or insufficiently appreciated. We investigate this phenomenon by looking at changes in the Dow Jones Industrial Average. Regression to the mean suggests that companies taken out of the Dow may not be as bad as their current predicament indicates and the companies that replace them may not be as terrific as their current record suggests. If investors are insufficiently aware of this statistical phenomenon, stock prices may be too low for the former and too high for the latter-mistakes that will be corrected when these companies regress to the mean. Thus, stocks taken out of the Dow may outperform the stocks that replace them. We test this hypothesis with the 50 substitutions made since the Dow expanded to 30 stocks in 1928.


## The Real Dogs of the Dow

The Dow Jones Industrial Average (the "Dow") contains 30 blue-chip stocks that represent the nation's most prominent companies. When companies falter, they are replaced by more successful companies. Regression to the mean suggests that companies doing poorly are more likely to have experienced bad luck than good, while the reverse is true of companies that have been doing well. If so, companies removed from the Dow may be stronger then they appear and the companies that replace them may not be as far above average as they seem. If investors are insufficiently aware of this statistical phenomenon, stock prices may be too low for the former and too high for the latter-mistakes that will be corrected when these companies regress to the mean. Thus, stocks taken out of the Dow may outperform the stocks that replace them. We test this hypothesis with the 50 substitutions made since the Dow expanded to 30 stocks in 1928.

## The Dow

In 1884, Charles H. Dow, the first editor of the Wall-Street Journal, calculated the average closing price for eleven stocks, nine of which were railroads. The industrial and rail averages were later separated and on October 1, 1928, the industrial average expanded to the current level of 30 firms that are intended to be "substantial companies-renowned for the quality and wide acceptance of their products or services-with strong histories of successful growth." (Dow Jones, 2006). The editors of the Wall-Street Journal periodically alter the composition of the Dow either to meet the index's objectives or to accommodate mergers or reorganizations.

## Index Effects

Several theories suggest that a company's inclusion in the S\&P 500 index may have a positive effect on the price of its stock. Because the S\&P 500 is a widely used target for index funds, a stock's addition to the S\&P 500 Index may have a substantial effect on the demand for the stock. Such shifts in demand will have little effect on a stock's price if demand is highly elastic, but may affect the price in the short run if investors must be compensated for accommodating demand
shifts (Harris and Gurel, 1986) or in the long run if the stocks in and out of the index are imperfect substitutes (Shleifer, 1989). Another possibility is that the addition of a stock to the S\&P 500 enhances a company's reputation and visibility, and this positive attention increases the price of the stock (Jain, 1987). Chen and Noronha (2002) argue that the prices of added stocks should increase permanently because of increased investor awareness of the stock, but that there is not a corresponding permanent decrease in the price of deleted stocks since investor awareness does not evaporate when a stock is deleted from the index. Amihud and Mendelson (1986) argue that a stock's inclusion in the S\&P 500 might reduce the bid-ask spread and that the diminished trading costs will increase the value of a stock.

There have been several empirical studies of changes in the S\&P 500 index, with inconsistent conclusions. Shleifer (1986), Harris and Gurel (1986), Jain (1987), and Beneish and Whaley (1996) all report short-term price increases for stocks added to the S\&P 500. Beneish and Whaley (2002) look at the period 1996-2001 and find short-term gains for stocks added to the S\&P 500 and short-term losses for deleted stocks. Lynch and Mendenhall (1997) look at the period 1990-1995 and find that added stocks experience price increases from the time of the announcement until the change is implemented and price declines after their inclusion; stocks taken out of the index experience price declines between the announcement and delisting and price increases after delisting. Cooper and Woglom (2002) find that the initial price increase for a stock added to the S\&P 500 is followed by a decrease due to the stock's increased volatility.

The Dow Jones Industrial Average should be much less susceptible to index effects since stocks added to the Dow are already well known and heavily traded, and the Dow average is not as popular as the S\&P 500 for indexing or derivatives. There have been relatively few studies of additions and deletions to the Dow. Polonchek and Krehbiel (1994) study changes in the Industrial and Transportation averages from 1962 to 1991-a period is characterized by 41 changes in the transportation index and 11 changes in the industrial index. They find positive
short-term abnormal returns and an increase in trading volume of the 11 firms that were added to the Industrial average but no abnormal returns for deleted firms. They find no significant abnormal returns for deletions or additions to the Transportation average. They argue that these results support Merton's (1987) hypothesis that news releases with no economic content may nonetheless draw investors' attention to a firm. Thus the price effects will be larger for additions than for deletions and will be larger for changes in the Industrial average because these are given more media coverage.

In contrast, Beneish and Gardner (1995) examine 37 changes in the Dow between 1929 and 1988 and find that firms added to the industrial average experience no change in price or trading volume but that deleted firms experience significant price declines. They argue that a stock's inclusion in the Dow does not significantly affect the stock because it is already actively traded. On the other hand, deleted firms are unpopular even before the deletion and their removal further reduces the likelihood that they will be followed by analysts. Ryan and Johnson (2001) find positive short-term (three days) abnormal returns for stocks added to the Dow and positive longterm (three years) abnormal returns for deleted stocks.

Earlier studies have been hampered by the fact that until very recently the Center for Research in Security Prices (CRSP) data base did not have daily stock returns prior to 1963, which compelled researchers to either limit their studies or collect their own data from newspaper microfiches, a tedious task subject to error. Because of the daunting task of assembling daily returns, those studies that do look at pre-1963 changes typically examine returns for only a few days surrounding the change.

In the present study, we look at daily returns over the entire period for all additions and deletions since 1928, when the Dow became an average of 30 stocks. Instead of looking at additions and deletions separately, we compare portfolios of the added and deleted stocks. Our expectation is that, contrary to previous studies, deleted stocks will outperform added stocks due
to regression to the mean.

## Regression to the Mean

Kahneman and Tversky (1973) argue that regression to the mean is a pervasive but subtle statistical principle that is often misunderstood or insufficiently appreciated:
$[R]$ egression effects are all about us. In our experience, most outstanding fathers have somewhat disappointing sons, brilliant wives have duller husbands, the ill-adjusted tend to adjust and the fortunate are eventually stricken by ill luck. In spite of these encounters, people do not acquire a proper notion of regression. First, they do not expect regression in many situations where it is bound to occur. Second, as any teacher of statistics will attest, a proper notion of regression is extremely difficult to acquire. Third, when people observe regression, they typically invent spurious dynamic explanations for it.

The educational testing literature provides a formal framework for explaining the statistical principle of regression to the mean. A person's observed test scores fluctuate about the unobserved latent trait measured by the test. This latent trait (the "true score") can be interpreted as the expected value of a person's test score, with the difference between a person's test score and true score called the "error score" (Lord and Novick, 1968). Among a group of test takers, those who score the highest are likely to have had positive error scores: it is possible, but unusual, for someone to score below his or her true score and still have the highest score on a test. Since a score that is high relative to the group is also likely to be high relative to that person's true score, this person's score on another test is likely to regress toward the mean.

This framework is directly applicable to a company's earnings. Actual earnings and predicted earnings both deviate from the probabilistic expected value of a company's earnings (its "true earnings"). Actual or predicted earnings that are high relative to a group of companies are also likely to be high relative to that company's true earnings. It is possible, but unlikely, that the most profitable company in 1998 had a negative error score that year, with earnings below its
expected value. It is possible, but unlikely, that the company predicted to be the most profitable in 1999 had a negative error score that year, with the prediction below the expected value of earnings.

We can consequently anticipate regression toward the mean when comparing consecutive earnings data or when comparing predicted and actual earnings. Freeman and Tse (1992) and Fama and French (2000) investigate the first question and find that successive earnings regress to the mean, although they attribute this regression to competitive forces rather than the purely statistical explanation that the error scores of companies with relatively high earnings are more likely to be positive than negative. Smith, Keil, and Smith (2004) look at the second question and find persuasive evidence that earnings forecasts are systematically too extreme-too optimistic for companies predicted to do well and too pessimistic for those predicted to do poorly. The accuracy of these forecasts can be improved consistently and substantially by shrinking them toward the mean forecast.

Lakonishok, Shliefer, and Vishny (1994) and La Porta (1996) provide additional evidence that investors do not fully understand regression to the mean. Using very different data, Vergin (2001) and Lee and Smith (2002) identify successful betting strategies based on the presumption that bettors do not fully appreciate regression to the mean in the performance of National Football League teams.

Here, if those who choose the stocks to include in the Dow do not understand regression to the mean, they may replace companies that are not in as dire straits as their recent performance suggests with companies that are not as stellar as they appear. Similarly, investors who are unaware of regression to the mean are likely to underestimate the strength of companies doing poorly and overestimate the strength of companies doing well, causing market prices to be too low for the former and too high for the latter relative to long-run fundamental values. When the performance of companies that had been doing poorly regresses to the mean, their stock prices
will correct upward; and when companies that had been doing well regress to the mean, their prices will fall. Thus, Smith, Keil, and Smith (2004) find that portfolios comprised of stocks with pessimistic earnings forecasts consistently outperform portfolios of stocks with more optimistic earnings forecasts. This argument suggests that a portfolio of stocks that have been deleted from the Dow may outperform a portfolio of stocks that replaced them.

## Data and Methodology

The CRSP data base now has daily returns back to January 2, 1926, which allow us to track daily returns on Dow additions and deletions since October 1, 1928, when the Dow was expanded from 20 to 30 stocks. The one exception is National Cash Register, which CRSP does not include until June 1, 1934. We compiled these daily returns from Wall Street Journal microfiches.

As of December 31, 2005, there have been 50 substitutions in the Dow. Table 1 lists these changes, which are typically announced on the day they are implemented. We excluded those changes in the Dow that were due to mergers, acquisitions, or name changes that did not involve the addition of one stock and deletion of another.

We created two portfolios: one of added stocks and one of deleted stocks. The portfolios include all stocks that have been added to or deleted from the Dow since October 1, 1928. The funds in the Deletion and Addition portfolios are invested equally in each of the companies previously added to or deleted from the Dow. The stocks in the portfolios are modified whenever another substitution is made in the Dow index or if a company disappears from the CRSP data base (because, for example, of a merger, takeover, or privatization). If a stock that had been previously deleted from the Dow is once again included in the Dow, it moves from the Deletion portfolio to the Addition portfolio; if a stock that is in the Addition portfolio is deleted from the Dow, it moves from the Addition portfolio to the Deletion portfolio. All changes are made at the end of the trading day on which the event occurs. Terminal wealth is as of December 31, 2005.

Table 2 summarizes the daily returns from these strategies. The two-sided p values are for a matched-pair test using the daily difference between the returns on the Deletion portfolio and the Addition portfolio. The null hypothesis is that the expected value of the difference in each day's return is zero. The $t$-statistic is

$$
\mathrm{t}=\frac{\mathrm{X}-0}{\mathrm{~s} / \sqrt{\mathrm{n}}}
$$

where $\overline{\mathrm{X}}$ is the mean of the daily differences, s is the standard deviation of the daily differences, and n is the number of daily differences.
. The Deletion portfolio beats the Addition by a margin that is both substantial and statistically persuasive. (Over 250 trading days, daily returns of 0.000588 and 0.000433 imply respective annual returns of $15.8 \%$ and $11.4 \%$.) Figure 1 shows that, with the exception of the 1990s, the Deletion portfolio consistently outperformed the Addition portfolio over the 76-year period.

For a simple test of whether this observed difference in returns is some sort of risk premium, we estimated the betas for each portfolio using the CRSP value-weighted index for the daily market returns:

$$
\begin{array}{rlr}
\text { Deletion: } \mathrm{R}=0.0190+1.0048 \mathrm{R}_{\mathrm{M}}, & \mathrm{R}^{2}=0.70 \\
{[3.99] \quad[219.15]} & \\
\text { Addition: } \mathrm{R}=0.0038+1.0233 \mathrm{R}_{\mathrm{M}}, & \mathrm{R}^{2}=0.80 \tag{0.74}
\end{array}
$$

where the $t$-values are in brackets. The Deletion-portfolio beta is close to 1 and slightly smaller than the Addition-portfolio beta.

We also estimated the Fama-French (1993) 3-factor model augmented by a momentum factor (Carhart 1997), though it is not certain that these factors (which were initially estimated for the period from July 1963 to December 1991) are relevant back to 1928 .

$$
\mathrm{R}=\alpha+\beta_{1} \mathrm{MKT}+\beta_{2} \mathrm{SMB}+\beta_{3} \mathrm{HML}+\beta_{4} \mathrm{UMD}+\varepsilon
$$

where
$\mathrm{R}=\quad$ return on the portfolio minus the one-month Treasury bill rate
MKT $=$ the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) minus the one-month Treasury bill rate (from Ibbotson)
$\mathrm{SMB}=$ average return on three small portfolios minus the average return on three big portfolios (size factor)
$\mathrm{HML}=$ the average return on two value portfolios minus the average return on two growth portfolios (book-to-market factor)
$\mathrm{UMD}=$ average return on two high prior return portfolios minus the average return on two low prior return portfolios (momentum factor)

This specification reflects the historical evidence that there are macro factors that cause stock returns to be positively correlated; small stocks tend to outperform big stocks (Banz 1981; Reinganum 1981); value stocks with high book-to-market ratios tend to outperform growth stocks (Rosenberg, Reid, and Lanstein, 1985); and stocks that have been doing well tend to outperform those doing poorly (Jegadeesh and Titman 1993). It is a unsettled whether these factors reflect risks that matter to investors (Chan 1988; Fama and French, 1992) or are evidence of market inefficiencies (Lakonishok, Shliefer, and Vishny, 1994). Either way, the question here is whether the relatively strong performance of the Deletion portfolio can be explained by these four factors.

All of the factor data were taken from Kenneth French's web site (2006). Because the daily factor data only go back to 1963, while the monthly factor data go back to 1926, we work with the monthly returns for our Deletion and Addition portfolios Table 3 shows the estimated equations. The substantial and statistically significant alpha values show that these four factors
do not explain the relatively strong performance of the Deletion portfolio. (Over 12 trading months, the annualized value of a $0.3050 \%$ daily excess return is $3.7 \%$.) The coefficient of the market factor is slightly above 1 ; the coefficients of the size and book-to-market factors are positive (the deleted stocks tend to be relatively small and lightly valued), and the momentum factor is negative. Interestingly, the Addition portfolio also has a positive alpha, though lower than the alpha for the Deletion portfolio. The coefficient of the market factor is slightly less than 1 and two of the other three factors are negative.

It is uncertain whether these Fama-French risk factors are the appropriate measure of risk for this entire period, but whether judged by these four factors or the simpler beta coefficients, it does not appear that the success of the Deletion portfolio can be attributed to the effects of the market, size, value, or momentum factors. Transaction costs will reduce the realized returns somewhat, but there remain large excess returns that are difficult to reconcile with the efficient market hypothesis.

The difference in returns is not due to the extraordinary performance of a few companies. Over this period, there were 50 additions and deletions. When more than one substitution occurred, we arbitrarily used the ordering reported on the Dow-Jones (2006) web site to match each addition with a deletion. We then calculated the returns for each of these matched pairs for as long as both stocks remained in the CRSP data base (for example, no bankruptcy or privatization) and neither stock was put back in or taken back out of the Dow. In 32 of 50 cases, the deleted stock did better than the added stock. If each deleted stock had a 0.50 probability of doing better than the matched added stock, the probability that 32 or more of 50 deleted stocks would do better than the added stocks is 0.0325 .

Another way to view the data is to average the returns across stocks, beginning on the day each substitution is made. Thus, we look at the daily returns for each deleted and added stock for ten years, beginning when the Dow substitution is made. After doing this for all substitution
dates, we average the returns on the first trading day after the substitution, on the second trading day after the substitution, and so on.

Table 4 summarizes the average levels of wealth for the Deletion and Addition stocks at 250day intervals (approximately 1 year) over the five-year period following the substitution dates. For example, the deleted stocks showed, on average, a $19.30 \%$ increase in value 250 trading days after the publication date, while the added stocks showed an average increase of only $3.37 \%$. The differences in average wealth grow increasingly pronounced as the horizon lengthens.

Figure 2 shows the ratio of the average deletion wealth to the average addition wealth each day over a ten-year horizon. The deleted stocks outpace the added stocks for approximately five years after the substitution date. Then their relative performance stabilizes.

## Conclusion

A portfolio consisting of stocks removed from the Dow Jones Industrial Average has outperformed a portfolio containing the stocks that replaced them. This finding contradicts the efficient market hypothesis since changes in the composition of the Dow are widely reported and well known. Our explanation for this anomaly is the market's insufficient appreciation of the statistical principle of regression to the mean, an error that has been previously identified in a variety of contexts and is no doubt present in a great many other contexts.

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Table 1 Dow Additions and Deletions After October 1, 1928

| January 8, 1929 | Additions | Deletions |
| :---: | :---: | :---: |
|  | National Cash Register | Victor Talking Machine |
| January 29, 1930 | Johns-Manville | North American |
| July 18, 1930 | Borden | American Sugar |
| May 26, 1932 | Eastman Kodak | American Tobacco B |
|  | Goodyear | Atlantic Refining |
|  | Liggett \& Myers | General Railway Signal |
|  | Standard Oil of California | Goodrich |
|  | United Air Transport | Nash Motors |
|  | Hudson Motor | Curtiss-Wright |
|  | American Tobacco B | Liggett \& Myers |
|  | Drug Incorporated | Mack Trucks |
|  | Proctor \& Gamble | United Air Transport |
|  | Loew's | Paramount Publix |
|  | Nash Motors | Radio Corporation |
|  | International Shoe | Texas Gulf Sulphur |
|  | International Business Machines | National Cash Register |
| August 15, 1933 | Coca-Cola | Hudson Motor |
|  | Corn Products Refining | Drug Incorporated |
|  | United Aircraft | International Shoe |
| August 13, 1934 | National Distillers | United Aircraft |
| November 20, 1935 | Du Pont | Borden |
|  | National Steel | Coca-Cola |
| March 14, 1939 | United Aircraft | Nash Kelvinator |
|  | American Telephone \& Telegraph | International Business Machines |
| July 3, 1956 | International Paper | Loew's |
| June 1, 1959 | Anaconda Copper | American Smelting |
|  | Swift \& Company | Corn Products Refining |
|  | Aluminum Company of America | National Steel |
|  | Owens-Illinois Glass | National Distillers |
| August 9, 1976 | Minnesota Mining \& Manufacturing | Anaconda Copper |


| June 29, 1979 | International Business Machines | Chrysler |
| :--- | :--- | :--- |
| August 30, 1982 | Merck | American Express |
| October 30, 1985 | McDonald's Corporation | Manville Corporation |
| March 12, 1987 | Coca-Cola | American Brands Incorporated |
|  | Boeing Company | Owens-Illinois |
| May 6, 1991 | Caterpillar Incorporated | Inco |
|  | Walt Disney Company | Navistar International |
| March 17, 1997 | J. P. Morgan \& Company | USX Corporation |
|  | Traveler's Group | Primerica Corporation |
|  | Jowlett-Packard Company | Westinghouse Electric |
|  | Wal-Mart Stores | Texaco Incorporated |
| November 1, 1999 | Microsoft | Bethlehem Steel |
|  | Intel | Woolworth |
|  | SBC Communications | Chevron |
|  | Home Depot | Goodyear Tire \& Rubber |
|  | American International Group | Union Carbide |
| April 8, 2004 | Pfizer | Sears, Roebuck |
|  | Verizon Communications | Eastman Kodak |
|  |  | International Paper |
|  |  |  |

Table 2 Daily Returns For Deletion and Addition Portfolios

| Deletion Portfolio |  | Addition Portfolio |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  |  |  |  | Diff-in-Means |
| Mean | Standard Deviation | Mean | Standard Deviation | P-value |
| 0.000588 | 0.012444 |  | 0.000433 | 0.011862 |

Table 3 Estimates of a Four-Factor Model, monthly returns (\%)

| Mean Excess |  |  |  |  |  |  | Adjusted |
| :--- | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Portfolio | Return | Alpha | MK T | SMB | HML | UMD | R-squared |
| Deletion | 1.059 | 0.305 | 1.020 | 0.187 | 0.406 | -0.103 | 0.85 |
|  |  | $[3.25]$ | $[55.72]$ | $[6.62]$ | $[14.87]$ | $[4.75]$ |  |
|  |  |  |  |  |  |  |  |
| Addition | 0.681 | 0.170 | 0.965 | -0.139 | 0.032 | -0.060 | 0.90 |
|  |  | $[2.80]$ | $[81.25]$ | $[7.62]$ | $[1.81]$ | $[4.30]$ |  |

t -values are in brackets

Table 4 Portfolio Wealth For Different Horizons after the Substitution Date

| Year | Day | Deletions | Additions |
| :---: | :---: | :---: | :---: |
| 1 | 250 | 1.1930 | 1.0337 |
| 2 | 500 | 1.5664 | 1.1499 |
| 3 | 750 | 1.5923 | 1.1455 |
| 4 | 1000 | 2.1925 | 1.4588 |
| 5 | 1250 | 2.7320 | 1.6504 |



Figure 1 Ratio of Deletion Portfolio Wealth to Addition Portfolio Wealth


Figure 2 Ratio of Deletion Portfolio Wealth to Addition Portfolio Wealth after Substitution

