Uncertainty and the adoption of complementary technologies

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In static models of the adoption of new technologies, a firm generally adopts complementary technologies simultaneously. This paper presents a dynamic model in which, taking into account the potential time paths of costs and benefits, it may be more prudent to adopt complementary technologies partially or sequentially.

1. Introduction

An important research question is how technological changes are adopted. Technology can be given a broad interpretation, including such innovations as the use of programmable machines, short-production cycles, short product development cycles, computer-aided design, barcoding, computer networks to collect, transmit and analyze data, and automated distribution systems. Many of these advances are complementary. The cost savings from using barcoding together with computer networks to transmit data, or automated distribution systems together with more frequent deliveries, or programmable production equipment together with computer-aided design are larger than the sum of the cost reductions from adopting either of the technologies alone. Jaikumar (1986, 1989), Brynjolfsson and Hitt (1993), and Colombo and Mosconi (1995) are examples of empirical studies documenting that many modern technologies are complementary.

It is generally assumed that a firm will adopt complementary technologies simultaneously. For example, the innovative comparative-statics model of Milgrom and Roberts (1990, 1995) implies that complementarities ‘make it relatively unprofitable to adopt only one part of the modern manufacturing strategy. The theory suggests that we should not see an extended period of time during which one component of the strategy is in place and the other components have barely begun to be put into place’ (Milgrom and Roberts, 1990: 524). Similarly, Jovanovic and Stolyarov (2000) write that ‘When a production process requires two extremely complementary inputs, conventional wisdom holds that a firm would always upgrade them simultaneously.’ They use nonconvex adjustment costs to upend the conventional wisdom. I use a dynamic model to do so.

One feature of many new technologies is that their costs decline rapidly. If the costs of complementary technologies are not anticipated to fall at the same rate, it may pay to adopt the technology whose price is falling more slowly and wait to adopt the
technology whose price is falling more rapidly. We also know from the dynamic programming literature that in an uncertain world, there is an option value to waiting to make investments that are costly to reverse (Jensen, 1982; Pindyck, 1988; Dixit, 1989). Even if costs are not certain to decline, uncertainty about the dynamic paths of either the benefits or the cost of the technology may be sufficient to warrant postponing some components of a package of complementary technologies.

Such a model can explain the reality that firms often adopt complementary technologies in stages. For example, Smith and Weil (2005) found that apparel manufacturers adopted barcoding technology first, followed by electronic data interchange (EDI) technology and then modular manufacturing practices. The static model used by Milgrom and Roberts implies revolutionary change in a revolutionary step. A dynamic model allows revolutionary change in evolutionary steps.

2. The model

A firm evaluating new technologies faces uncertainty regarding both the potential cost savings from adopting a new technology and changes in the cost of this technology. The firm must weigh the benefits from implementing the new technology immediately against the costs of doing so, including the cost of exercising its adoption option and thereby forfeiting the benefits from waiting for more information about the evolution of cost savings and technology costs.

Assume that the evolution of annual operating costs $C$ can be described by this geometric Brownian motion equation:

$$\frac{dC}{C} = \alpha dt + \sigma dz$$

(1)

where $\alpha$ is the trend rate of growth of costs and $dz$ is the increment of a standard Wiener process.

I consider here two complementary technologies. One example is bar codes and EDI. Bar codes provide a means for receiving detailed demand information; EDI provides a method of frequent and accurate transmission of demand data. Firms that adopt both bar codes and EDI are able to reduce the transaction costs for processing shipments since this combination enables retailers to scan incoming shipments, check them against purchase orders, authorize payments to suppliers, and rapidly identify discrepancies between invoices and actual shipments. Another example is automated shipping and modular manufacturing practices. Automated shipping technology enables firms to move products quickly from the supplier to the buyer; modular production methods enable firms to reduce the time required for a given product to move through the assembly process. Firms that adopt both automated shipping technologies and modular production methods are able to reduce throughput time more effectively since this combination enables suppliers to distribute the finished products more quickly. A third example is information technology and modular production.
The suite of information technologies that enable firms to capture, use and convey accurate demand information complements modular production methods because it enhances the firm’s ability to respond to timely demand information effectively.

In my model, the use of Technology 1 (e.g. information technologies) alone provides net cost savings \( S_1 \) that are a fraction \( \lambda_1 \) of operating costs: \( S_1 = \lambda_1 C \). Because these potential savings are proportional to operating costs, they evolve according to the same Brownian motion equation:

\[
\frac{dS_1}{S_1} = \alpha dt + \sigma dz
\]

Similarly, the use of Technology 2 (e.g. modular manufacturing methods) alone provides cost savings \( S_2 = \lambda_2 C \), with

\[
\frac{dS_2}{S_2} = \alpha dt + \sigma dz
\]

The simultaneous use of both technologies yields cost savings \( S = \lambda C \), with

\[
\frac{dS}{S} = \alpha dt + \sigma dz
\]

If the technologies are complementary, then \( \lambda > \lambda_1 + \lambda_2 \).

The respective costs \( K_1 \) and \( K_2 \) of adopting these two technologies also evolve according to geometric Brownian motion:

\[
\frac{dK_1}{K_1} = \alpha_1 dt + \sigma_1 dz_1
\]

\[
\frac{dK_2}{K_2} = \alpha_2 dt + \sigma_2 dz_2
\]

The relevant correlation coefficients between operating costs \( C \) and adoption costs \( K_1 \) and \( K_2 \) are \( \rho_{C1} \) and \( \rho_{C2} \). The covariance between the two adoption costs is \( \rho_{12} \).

A two-stage process can be used to analyze a firm’s decision to adopt, say, Technology 1. The cost is \( K_1 \) and the expected present value \( V_1 \) of the annual cost savings \( \lambda_1 C \) is

\[
V_1 = \frac{\lambda_1 C}{\mu - \alpha}
\]

where \( \mu \) is the risk-adjusted required rate of return. It will often be convenient to work with \( \delta = \mu - \alpha, \delta_1 = \mu_1 - \alpha_1 \) and \( \delta_2 = \mu_2 - \alpha_2 \), where \( \mu_1 \) and \( \mu_2 \) are the risk-adjusted required returns for Technologies 1 and 2.

In addition to the annual cost savings, the adoption of Technology 1 gives the firm the option to adopt Technology 2 also, which will cost \( K_2 \) and provides this additional present value:
3. Adoption thresholds with no uncertainty

If there is no uncertainty (\(\sigma_1 = \sigma_2 = 0\)), then the operating cost and adoption cost functions can be written as

\[
C[t] = C[0]e^{\mu t}
\]

\[
K_1[t] = K_1[0]e^{\alpha_1 t}
\]

\[
K_2[t] = K_2[0]e^{\alpha_2 t}
\]

Using Equation 4, the current net present value of adopting Technology 1 alone at time t is

\[
P_1[t] = (V_1[t] - K_1[t])e^{-\mu t}
\]

\[
= \left(\frac{\lambda_1 C[t]}{\mu - \alpha} - K_1[t]\right)e^{-\mu t}
\]

\[
= \frac{\lambda_1 C[0]}{\mu - \alpha} e^{(\alpha_1 - \mu)t} - K_1[0]e^{(\alpha_1 - \mu)t}
\]

The first-order condition is that the percentage change in the net present value is equal to the discount rate:

\[
\frac{d(V_1[t] - K_1[t])}{dt} \quad \frac{1}{V_1[t] - K_1[t]} = \mu
\]

For the parameterization used here,

\[
\frac{dP_1[t]}{dt} = (\alpha - \mu)\frac{\lambda_1 C[0]}{\mu - \alpha} e^{(\alpha_1 - \mu)t} - (\alpha_1 - \mu)K_1[0]e^{(\alpha_1 - \mu)t}
\]

\[
= 0 \text{ at } t = t^* \text{ when }
\]

\[
e^{(\alpha - \alpha_1)t^*} = \frac{(\mu - \alpha_1)K_1[0]}{\lambda_1 C[0]}
\]

\[
t^* = \frac{\ln\left(\frac{(\mu - \alpha_1)K_1[0]}{\lambda_1 C[0]}\right)}{\alpha - \alpha_1}
\]

This condition can be rephrased in terms of an adoption threshold; that is, adopt Technology 1 alone when the ratio of operating costs to adoption cost reaches this level:
Using a similar procedure, the optimal time to adopt Technology 2 if Technology 1 has already been adopted is when this threshold is reached:

\[
\left( \frac{C}{K_1^*} \right) = \frac{C[0]e^{\alpha t^*}}{K_1[0]e^{\alpha t^*}} = \left( \frac{\mu - \alpha_1}{\lambda_1} \right) \left( \frac{\mu - \alpha_1}{\lambda_1} \right) K_1[0] \frac{e^{\alpha t^*}}{K_1[0]} \frac{\mu - \alpha_1}{\lambda_1} C[0] = \frac{\mu - \alpha_1}{\lambda_1}
\]

The thresholds for adopting Technology 2 alone and for adopting Technology 1 if Technology 2 has been adopted are given by given by analogous equations, simply interchanging the 1 and 2 subscripts.

### 4. Adoption thresholds with uncertainty

If there is uncertainty and it is costly to undo investments (at the extreme, investments are irreversible), then the simple investment rules given in Section 3 are inapplicable. If the operating-cost savings turn out to be smaller than projected, the firm may regret having invested in new technologies. If the cost of the adopting new technologies declines more than projected, the firm may wish it had waited to adopt. Thus, there is a potential benefit from waiting for savings-cost conditions that are even more decisively favorable or unfavorable than those currently prevailing.

Dynamic programming can be used to determine the value \( F_{21}[C, K_2] \) of the option to adopt Technology 2 when Technology 1 is in place. Once we have this, we can determine the total value \( V_1 + F_{21} \) from adopting Technology 1 alone.

Setting the return on the option equal to the expected capital gain on the option and using Ito’s lemma, we obtain this differential equation for the region in which the firm waits to invest:

\[
0 = 0.5 \left\{ \sigma^2 C^2 \frac{\partial^2 F_{21}}{\partial C^2} + 2\rho_{C2} \sigma \sigma_2 C K_2 \frac{\partial^2 F_{21}}{\partial C \partial K_2} + \sigma_2^2 K_2^2 \frac{\partial^2 F_{21}}{\partial K_2^2} \right\} + (r - \delta) \frac{\partial F_{21}}{\partial C} + (r - \delta_2) K_2 \frac{\partial F_{21}}{\partial K_2} - r F_{21}
\]
A natural assumption that makes the analysis tractable is that the option value is homogeneous of degree one: \( F_{21}(C, K_2) = K_2 f_{21}(C/K_2) \). The substitution of the requisite partial derivatives into our differential equation yields

\[
0 = 0.5 \phi_2 (C/K_2)^2 f'_{21} + (\delta_2 - \delta)(C/K_2) f_{21} - \delta_2 f_{21}
\]

where

\[
\phi_2 = \sigma^2 - 2 \rho_{C2} \sigma \sigma_2 + \sigma_2^2
\]

This is a homogeneous second-order linear differential equation whose general solution has the form

\[
f_{21}(C/K_2) = A_{21}(C/K_2)^{b_2} + B_{21}(C/K_2)^{c_2}
\]

where

\[
b_2 = 0.5 + \frac{\delta - \delta_2}{\phi_2} + \frac{0.5 + \delta - \delta_2}{\phi_2} \frac{2 \delta_2}{\phi_2} > 1
\]

\[
c_2 = 0.5 + \frac{\delta - \delta_2}{\phi_2} - \frac{0.5 + \delta - \delta_2}{\phi_2} \frac{2 \delta_2}{\phi_2} < 0
\]

As the ratio of operating costs to adoption cost, \( C/K_2 \), approaches 0, the value of the option does too; therefore, \( B_{21} = 0 \). We can use the value-matching and smooth-pasting conditions at the threshold ratio \( R_{21} = (C/K_2)^* \), where the firm is indifferent about exercising its option to obtain

\[
R_{21} = \frac{b_2}{b_2 - 1} \frac{\delta}{\delta_1}
\]

Substitution into equation (5) shows that the optimal investment rule is not \( V_{21} > K_2 \), but rather \( V_{21} > (b_2/(b_2 - 1))K_2 \), which is larger than \( K_2 \) since \( b_2 \) is larger than 1. The investment costs \( K_2 \) and yields an anticipated cash flow currently worth \( V_{21} \), but also extinguishes the firm’s option to postpone the adoption decision to a time when it may be even more profitable or it may be evident that the cash flow has turned out to be disappointing.

It can be shown that the threshold ratio depends on these underlying parameters:

\[
(C/K_2)^* = R_{21}[\mu, \lambda - \lambda_1, \alpha, \alpha_2, \sigma, \sigma_2, \rho_{C2}]
\]

Ceteris paribus, the threshold is clearly higher if there is a higher required rate of return, lower cost savings, or lower growth rate of cost savings. A higher growth rate of the technology’s cost reduces the threshold since it is better to invest sooner if the
cost is rising and better to wait if the cost is falling. The larger the standard deviation of operating costs or technology costs, the higher is the threshold, because of the increased value of postponing the decision and keeping alive the option to invest. A larger covariance between operating and technology costs reduces the variance of their ratio and consequently reduces the value of waiting.

Now that we have the value of the implicit option on Technology 2 if Technology 1 has been adopted,

\[ F_{21} = K_2 A_{21} (C/K_2)^{b_2} \quad \text{if} \quad C/K_2 \leq R_{21}^{*} \]

\[ = \frac{V_{21}}{K_2} - 1 \quad \text{otherwise} \]

we can analyze the first-stage decision to adopt Technology 1.

The expected present value of the annual cost savings from adopting Technology 1 is:

\[ V_1 = \frac{\lambda_1 C}{\mu - \alpha} \]

Let \( F[C, K_1, K_2] \) be the value of the option to adopt either or both technologies. Setting the return on the option \( rF \) equal to the expected capital gain on the option and using Ito’s lemma, we obtain this differential equation for the region in which the firm waits to invest:

\[ 0 = 0.5 \left\{ \sigma_1^2 C^2 \frac{\partial^2 F}{\partial C^2} + \sigma_1^2 K_1^2 \frac{\partial^2 F}{\partial K_1^2} + \sigma_2^2 K_2^2 \frac{\partial^2 F}{\partial K_2^2} + 2 \rho_{12} \sigma_1 \sigma_2 K_1 K_2 \frac{\partial^2 F}{\partial K_1 \partial K_2} \right\} \]

\[ + (r - \delta) C \frac{\partial F}{\partial C} + (r - \delta_1) K_1 \frac{\partial F}{\partial K_1} + (r - \delta_2) K_2 \frac{\partial F}{\partial K_2} - rF \]

In the region where the firm is waiting to adopt, this value can be separated into the value of the option to acquire Technology 1 plus the value of the option to acquire Technology 2 as well. I again assume first-order homogeneity,

\[ F[C, K_1, K_2] = K_1 f_{1}(C/K_1) + K_2 f_{21}(C/K_2) \]

The relevant partial derivatives yield

\[ 0 = (0.5 \phi_1 (C/K_1) f_1' + (\delta_1 - \delta) f_1' - \delta_1 (K_1/C) f_1) \]

\[ + (0.5 \phi_2 (C/K_2) f_{21}' + (\delta_2 - \delta) f_{21}' - \delta_2 (K_2/C) f_{21}) \]
where
\[ \phi_1 = \sigma^2 - 2 \rho_{C_1} \sigma \sigma_1 + \sigma_1^2 \]

In the region where \( C/K_2 \leq R_{21} \), the second bracketed expression is equal to zero, leaving this second-order linear differential equation:

\[ 0 = 0.5 \phi_1 (C/K_1) f_1'' + (\delta_1 - \delta) f_1' - \delta_1 (K_1/C) f_1 \]

The economically meaningful solution is

\[ f_1[C/K_1] = A_1(C/K_1)^{b_1}, \]

where

\[ b_1 = 0.5 + \frac{\delta - \delta_1}{\phi_1} + \left( \frac{0.5 + \frac{\delta - \delta_1}{\phi_1}}{\phi_1} \right)^2 + \frac{2 \delta_1}{\phi_1} > 1 \]

The value-matching and smooth-pasting conditions at the threshold ratio \( R_1 = (C/K_1)^* \) and some algebraic manipulation yield

\[ R_1 = \frac{b_1}{b_1 - 1} \frac{\delta}{\lambda_1} \]

For any given value of \( C/K_2 \), this threshold ratio depends on these underlying parameters:

\[ \left( \frac{C}{K_1} \right)^* = R_1[\mu, \lambda_1, \alpha, \alpha_1, \sigma, \sigma_1, \rho_{C_1}] \tag{7} \]

The parameters are very similar to those in equation (6) for the adoption of Technology 2 if Technology 1 has been adopted. Ceteris paribus, the threshold for adopting Technology 1 is higher if there is a higher required return, lower cost savings, lower growth rate of either the cost savings or the cost of Technology 1, higher standard deviation of operating costs or technology costs, and a larger covariance between operating and technology costs.

Notice that the threshold for adopting Technology 1 does not depend on the value of \( C/K_2 \). When the firm has not adopted either technology, it holds an option with value \( F \) that depends on two factors: the option to use Technology 1 and the option to use Technology 2 in addition to Technology 1. If the firm does adopt Technology 1, it will still have the option to use Technology 2 too, and the value of this option has not been affected by the adoption of Technology 1. The firm has simply cashed in its option to use Technology 1, giving it a profit of

\[ V_1 = K_1 f_1[C/K_1] - K_1 \]
To analyze the firm’s decision to adopt Technology 2 first, possibly followed by Technology 1, we can simply interchange the subscripts. For example, the threshold for adopting Technology 1 if Technology 2 has been adopted is

$$(C/K_1)^* = R_{12} = \frac{b_1}{b_1 - 1} \frac{\delta}{\lambda - \lambda_2}$$

and the cutoff for adopting Technology 2 by itself is

$$(C/K_2)^* = R_2 = \frac{b_2}{b_2 - 1} \frac{\delta}{\lambda_2}$$

An interesting case is where, instead of being complements, the two cost-saving technologies have independent effects: $\lambda = \lambda_1 + \lambda_2$. Now, the threshold for adopting Technology 1 when Technology 2 is already adopted is identical to the threshold for adopting Technology 1 alone:

$$R_{12} = \frac{b_1}{b_1 - 1} \frac{\delta}{\lambda - \lambda_2} = \frac{b_1}{b_1 - 1} \frac{\delta}{\lambda_1} = R_1$$

Similarly, the threshold for adopting Technology 2 when Technology 1 is already adopted is identical to the threshold for adopting Technology 2 alone:

$$R_{21} = \frac{b_2}{b_2 - 1} \frac{\delta}{\lambda - \lambda_2} = \frac{b_2}{b_2 - 1} \frac{\delta}{\lambda_2} = R_2$$

When the technologies are complements, the degree of complementarity does not affect the decision to adopt either technology by itself, but does reduce the threshold for adopting the other if one technology is adopted.

5. Illustrative parameter values

To illustrate this model, consider the following parametric assumptions: $\lambda_1 = 0.10$, $\lambda_2 = 0.10$, $\lambda = 0.30$, $\mu_1 = \mu_2 = 0.10$, $\alpha_1 = 0.05$, $\sigma_1 = 0.05$, $\alpha_2 = 0.05$, $\sigma_2 = 0.10$, $\sigma_1 = 0.20$, $\rho_{C_1} = 0.0$, and $\rho_{C_2} = 0.0$. Either technology alone will reduce costs by 10%; both together will reduce costs by 30%. The required return on operating cost and both technology costs is 10 percent. Operating costs have a 5% trend growth rate and 5% standard deviation; the cost of Technology 1 has a $-5\%$ trend growth rate and 5% standard deviation; the cost of Technology 2 has a $-10\%$ trend growth rate and 20% standard deviation. Changes in technology costs are uncorrelated with operating costs.

The crucial distinction between the two technologies is that the cost of Technology 2 is expected to fall more rapidly and has a higher standard deviation, both of which increase the value of the option to adopt this technology at a later date. Figure 1 shows the four adoption thresholds. As anticipated, Technology 2 has higher adoption thresholds, either by itself or after the adoption of the other technology, because the
The cost of adopting Technology 2 is declining more rapidly and has a higher standard deviation, which are each incentives for waiting. Notice also that because these technologies are complementary, the threshold ratio for each technology is approximately 50% lower when the other technology has been adopted.

Asynchronous adoption does not require an anticipated decline in technology costs; uncertainty about future adoption costs is sufficient. Table 1 shows the four adoption thresholds if the two cost trend parameters are changed to \( \alpha_1 = \alpha_2 = 0 \), and if all three trend parameters are set equal to zero: \( \alpha = \alpha_1 = \alpha_2 = 0 \). In each case, the adoption threshold for Technology 2, either alone or if the other technology has been adopted, is higher than for Technology 1 because of the greater uncertainty about the future cost of adopting Technology 2.

**Table 1** Adoption thresholds for three scenarios

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( R_1 = (C/K_1))</th>
<th>( R_{12} = (C/K_1))</th>
<th>( R_2 = (C/K_2))</th>
<th>( R_{21} = (C/K_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>-0.05</td>
<td>-0.10</td>
<td>1.54 (1.50)</td>
<td>0.77 (0.75)</td>
<td>2.27 (2.00)</td>
<td>1.12 (1.00)</td>
</tr>
<tr>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>1.05 (1.00)</td>
<td>0.52 (0.50)</td>
<td>1.34 (1.00)</td>
<td>0.67 (0.50)</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.17 (1.00)</td>
<td>0.59 (0.59)</td>
<td>1.58 (1.00)</td>
<td>0.79 (0.50)</td>
</tr>
</tbody>
</table>

Figures in parentheses are thresholds with no uncertainty.
Table 1 also shows parenthetically the adoption thresholds if there is no uncertainty. These calculations demonstrate that the presence of a plausible degree of uncertainty can have a dramatic effect on the adoption thresholds, here raising the requisite ratio of operating costs to adoption cost by up to 58%.

6. Implications

This paper presents a dynamic model of the adoption of complementary technologies, where the incentive to invest is affected by uncertain projections of costs and benefits. In general, firms are more willing to delay the adoption of some or all technologies if the cost of the technology is expected to decline, if there is considerable uncertainty about the evolution of costs or benefits, or if the discount rate is high. The time to adoption will turn out to be delayed or never occur if the cost of adopting the technology remains high or the potential cost savings turn out to be disappointing.

If firms are heterogeneous, firms will adopt at different times and not necessarily in the same order. The first movers adopt first because their anticipated cost savings are sufficiently high relative to the adoption costs. For example, the unique factors that affect a firm’s cost savings from new information technology include the firm’s size, product variety, inventory holding costs and usage of complementary technologies. Size matters because the adoption of new technologies will affect a larger set of products, yielding larger economies of scale in the adoption benefits. Product variety matters because the adoption of information technology allows firms to increase their ability to track the variability in sales across a wider array of products and reduce their dependency on manual counting that is both more time consuming and prone to errors. Cost savings might also be expected to be larger for firms with high inventory holding costs because the introduction of new information technology systems enables them to increase their knowledge about the state of demand and reduce their dependency on inventories by allowing them to produce more to demand. Lastly, the cost savings might also be expected to be higher for firms that have already adopted complementary technologies such as changes in manufacturing, order processing, sales and marketing. The introduction of new information technologies will induce greater cost savings for firms that have already streamlined their order processing practice and now manufacture in flexible small batches, because they can more quickly respond to new and timely information than can firms that have not made these technological adaptations. Firms that have already adopted complementary practices will be able to benefit from lower stockouts and lower inventory holding costs than firms that have not.

In a world where the dynamic time paths of costs and benefit matter, we should not be surprised to find asynchronous technology adoption within firms and across firms.
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