The Intrinsic Value of Owner-Occupied Housing

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#### Abstract

Residential real estate is commonly valued by using "comps," the prices for recent transactions of comparable homes. A focus on comps can fuel speculative booms-in Beanie Babies, dot-com stocks, and real estate. A very different way to value houses is by determining the present value of the anticipated cash flow. This is a very robust and well-established procedure that is widely used to value bonds, stocks, business projects, and commercial real estate, and it is striking how seldom it is used to value residential real estate.


## The Intrinsic Value of Owner-Occupied Housing

Residential real estate is commonly valued by looking at "comps," the prices for recent transactions of comparable homes. The comps may be analyzed informally by realtors or systematically by multiple regression models and other statistical techniques (for example, Isakson 1998; Detweiler and Radigan 1996, 1999; Nguen and Cripps 2001). A focus on comps can fuel speculative booms. In 1972, it seemed okay to buy MGIC stock at 83 times earnings because Polaroid traded at 90 times earnings. In 1998 it seemed okay to pay $\$ 500$ for a Britannia the British Bear Beanie Baby because a Princess Beanie Baby sold for \$500. In 2001 it seemed okay to pay 800 times revenue for a dot-com company because another traded at 900 times revenue. Today it seems okay to pay $\$ 2$ million for modest 2000-2500 square-foot houses in Palo Alto, California because a tear-down property sold for more than $\$ 2$ million.

A very different way to values houses is by determining the present value of the anticipated cash flow. This is a robust and well-established procedure that is widely used to value bonds, stocks, business projects, and commercial real estate. It is surprising that it is not routinely used to value residential real estate.

## Speculative Fuel

One possible reason for focusing on comps is a belief that a house is a foolproof investment. Theodore Roosevelt advised that, "Every person who invests in well selected real estate in a growing section of a prosperous community adopts the surest and safest method of becoming financially independent." Barbara Alexander, author of "The Other Side of the Coin," says of residential real estate: "there is no bad time to buy" (Alexander 2003). If there is never a bad time to buy, then a buyer's primary concern is that a house's price is reasonable in comparison to the prices of other houses, not whether the price is justified in any absolute sense.

A superficially substantive justification for the idea that real estate is a foolproof investment is
an appeal to the laws of supply and demand. Thus the advice of humorist Will Rogers is cited: "Don't wait to buy land, buy land and wait, the good Lord ain't makin' any more of it." Variations involve the observations that much of the value of residential real estate is the land the house sits on, and there is a fixed supply of land and a growing demand for housing due to increases in the size of the population, or the average age of the population, or the wealth of the population (for example, Joint Center for Housing Studies of Harvard University 2002). But is this logic really compelling? The same argument could be made about anything with a fixed supply, many of which would not be considered seriously as investments: rocks, Chrysler New Yorkers, last year's clothing fashions.

Some people look at how much housing prices have increased in the past and assume that housing prices will increase at that rate in the future (Conti and Finkel 2002, Eldred 2001, Roos 2001). Such incautious extrapolation is speculation, not investment. To investors, an asset is worth the present value of its prospective cash flow discounted by a required return that takes into account the returns available on alternative investments, its risk, and other salient considerations. To speculators, an asset is worth what someone else will pay for it, and the challenge is to guess what others will pay tomorrow for what you buy today. This guessing game is dangerously close to the Greater Fool Theory: you buy something at an inflated price, hoping to find an even bigger fool who will buy it from you at a still higher price. People who bought Beanie Babies and dot-com stocks at their peaks learned the expensive lesson that past price increases are no guarantee of future price increases.

For an investment, the crucial issue is not whether the supply is fixed or how much higher the price is today than yesterday, but whether the cash flow generated by the asset justifies the price. It doesn't matter how scarce a Beanie Baby is or how much its price has increased; if it doesn't generate cash or an equivalent, it may be an interesting (and temporarily profitable)
speculation, but it is a worthless investment.

## The Rent Alternative

The cash flow from owner-occupied residential real estate is the rental payments the owner would otherwise have to pay. One alternative to buying dot-com stocks and Beanie babies was bonds. An alternative to buying a house is renting one. Some authors recognize this alternative, but simply list the pluses and minus of buying and renting (Goodman 2001, Miller 2002, Orman 2001). Others compare the monthly mortgage payments with the monthly rent for a similar property. For example, in its 2002 housing report, the Joint Center for Housing Studies of Harvard University estimates that in 2001 the average renter paid \$481/month while the purchaser of a median single-family home incurred $\$ 821$ of after-tax mortgage payments. This approach is obviously flawed. The median single-family home is unlikely to be equivalent to the average rental property. Even if it were, monthly mortgage payments depend on the size of the downpayment and the length of the mortgage. Suppose, for an extreme example, someone paid cash for a house and had no mortgage payments. Does that mean buying is better than renting? Another problem is that their calculations don't consider that rents increase over time while mortgage payments are constant and end when the mortgage is paid off.

Quinn (1997) compares the costs of buying and renting taking into account rent increases, but ignores property taxes, utilities, maintenance and other expenses and assumes a 7-year holding period. Personal finance textbooks are not much better. Keown (1998) uses a worksheet with a 7year horizon that neglects present value and rent increases. Ramaglia and MacDonald (1999) do the same, but additionally ignore the equity component of mortgage payments, the appreciation in the value of the house, and the selling costs. Winger and Fresca (2000) use a 5-year horizon that neglects present value and ignores the equity component of mortgage payments. Kapoor, Dlabay, and Hughes (1999) only look at the first year and ignore the fact that rent and various
expenses will increase in the future.
UCLA economist Edward Leamer recommends, analogous to stocks, looking at a house's P/E: the ratio of the house's market value to its annual rental value. An article describing this approach says "it's the change in P/E that matters-not the number" (Feldman 2003). An accompanying table looks for evidence of a housing bubble by showing that housing prices increased more than rents between 1993 and 2002 and identifies the cities in which the $\mathrm{P} / \mathrm{E}$ increased the most. In reality, it is the number that matters, not the change. If we are choosing between buying and renting today, what matters is housing prices and rents today.

Just as with stocks, there are valid reasons for $\mathrm{P} / \mathrm{Es}$ to go up and down and for different assets to have different P/Es. If interest rates fall, as they did between 1993 and 2002, the P/Es for stocks, houses, and other assets should rise. Companies with rapidly growing earnings and houses with rapidly growing rents should have higher P/Es than slow-growth stocks and slowgrowth houses. In fact, the article's table shows that the housing markets with the strongest $\mathrm{P} / \mathrm{Es}$ are those in which rents were increasing the most rapidly.

Leamer's fundamental principle-that houses can be valued the way stocks are valued-is valid. However, just as with stocks, we need a model to project future earnings and, just as with stocks, the $\mathrm{P} / \mathrm{E}$ is suggestive, but not definitive.

## Intrinsic Value

We can value stocks the same way we value bonds, stocks, and other assets-by calculating the present value of the cash flow. Consider first the unlikely case where you pay cash for a house, the same way that you might pay cash for a stock. The intrinsic value V is the present value of the cash flow $X_{t}$, discounted by the required rate of return $R$ :

$$
\begin{equation*}
\mathrm{V}=\frac{\mathrm{X}_{1}}{(1+\mathrm{R})^{1}}+\frac{\mathrm{X}_{2}}{(1+\mathrm{R})^{2}}+\frac{\mathrm{X}_{3}}{(1+\mathrm{R})^{3}}+\ldots \tag{1}
\end{equation*}
$$

The implicit income from a house is the rent you would otherwise have to pay to live in this house minus the expenses associated with home ownership. If you would pay $\$ 30,000$ a year in rent, home ownership implicitly gives you $\$ 30,000$ that you otherwise would pay to someone else. On the other hand, as a homeowner, you must pay property taxes, insurance, maintenance, and some utilities that you would not have to pay if you were a renter. If these expenses are $\$ 10,000$ a year, your implicit net annual income is $X_{1}=\$ 30,000-\$ 10,000=\$ 20,000$.

To determine the present value, we must project this implicit income into the future. As with stocks, the constant growth model provides a simple and insightful starting point. If the cash flow grows as a rate $\mathrm{g}<\mathrm{R}$, so that $\mathrm{X}_{\mathrm{t}+1}=(1+\mathrm{g}) \mathrm{X}_{\mathrm{t}}$, then the present value formula simplifies to

$$
\begin{equation*}
\mathrm{V}=\frac{\mathrm{X}_{1}}{\mathrm{R} \square \mathrm{~g}} \tag{2}
\end{equation*}
$$

For example, if the cash flow is $\$ 20,000$, growing at $5 \%$ a year, and the required rate of return is $10 \%$, this house's intrinsic value is $\$ 400,000$ :

$$
\begin{aligned}
\mathrm{V} & =\frac{\mathrm{X}_{1}}{\mathrm{R} \square \mathrm{~g}} \\
& =\frac{\$ 20,000}{0.10 \square 0.05} \\
& =\$ 400,000
\end{aligned}
$$

One attractive feature of this model is its internal consistency. The current $\$ 20,000$ cash flow on a house valued at $\$ 400,000$ provides $5 \%$ in current income: $\$ 20,000 / \$ 400,000=0.05$. If the cash flow grows, as projected, by $5 \%$ a year, then the present value will also grow by $5 \%$ a year. Together, the $5 \%$ current income and the $5 \%$ growth in value provide the requisite $10 \%$ return.

A homebuyer can use the projected cash flow and a required rate of return to determine if a house's intrinsic value is above or below its market price P . If the price is less than or equal to the intrinsic value, the house is indeed worth what it costs. If the market price is above intrinsic
value, renting is more financially attractive.
Equivalently, we can calculate the net present value (NPV): the intrinsic value minus the price: $\mathrm{NPV}=\mathrm{V}-\mathrm{P}$. More generally, we can modify Equation 1 so that the NPV is the present value of the entire cash flow, including the purchase price:

$$
\begin{equation*}
\mathrm{NPV}=\mathrm{X}_{0}+\frac{\mathrm{X}_{1}}{(1+\mathrm{R})^{1}}+\frac{\mathrm{X}_{2}}{(1+\mathrm{R})^{2}}+\frac{\mathrm{X}_{3}}{(1+\mathrm{R})^{3}}+\ldots \tag{3}
\end{equation*}
$$

If the homebuyer pays cash, $\mathrm{X}_{0}$ is a negative number equal to the price. Later in this paper, when we introduce mortgages, $\mathrm{X}_{0}$ will be a negative number equal to the downpayment and out-ofpocket closing costs.

The intrinsic-value model in Equation 1 implicitly assumes that the investor is buying "for keeps." One reflection of the model's internal consistency is that if the investor can sell the asset at some future date for a price equal to its intrinsic value at that time, the current intrinsic value of the cash flow and sale price is equal to the intrinsic value of the cash flow if the investor holds for keeps. Suppose the investor sells in period $n$ for a price $P_{n}$ that is equal to the present value of the cash flow beyond period n :

$$
\begin{equation*}
P_{n}=\frac{X_{n+1}}{(1+R)^{1}}+\frac{X_{n+2}}{(1+R)^{2}}+\ldots \tag{4}
\end{equation*}
$$

The current present value of the cash flow and sale price is

$$
\begin{equation*}
\mathrm{V}=\frac{\mathrm{X}_{1}}{(1+\mathrm{R})^{1}}+\frac{\mathrm{X}_{2}}{(1+\mathrm{R})^{2}}+\ldots+\frac{\mathrm{P}_{\mathrm{n}}}{(1+\mathrm{R})^{n}} \tag{5}
\end{equation*}
$$

The substitution of Equation 4 into Equation 5 gives the same intrinsic value as in Equation 1, when the asset is bought for keeps:

$$
\begin{aligned}
\mathrm{V} & =\frac{\mathrm{X}_{1}}{(1+\mathrm{R})^{1}}+\frac{\mathrm{X}_{2}}{(1+\mathrm{R})^{2}}+\ldots+\frac{1}{(1+\mathrm{R})^{n}} \frac{\square}{\square} \frac{X_{n+1}}{-(1+R)^{1}}+\frac{\mathrm{X}_{\mathrm{n}+2}}{(1+\mathrm{R})^{2}}+\ldots \\
& =\frac{\mathrm{X}_{1}}{(1+\mathrm{R})^{1}}+\frac{\mathrm{X}_{2}}{(1+\mathrm{R})^{2}}+\ldots+\frac{\mathrm{X}_{\mathrm{n}+1}}{(1+\mathrm{R})^{n+1}}+\frac{\mathrm{X}_{\mathrm{n}+2}}{(1+\mathrm{R})^{n+2}}+\ldots
\end{aligned}
$$

## A Leveraged Investment

In practice, matters are more complex because, unlike stocks, real estate is typically a highly leveraged investment financed mostly with a mortgage and this leverage matters if the mortgage rate is not equal to the buyer's required rate of return.

To see this, suppose that the buyer makes a downpayment equal to a fraction $\square$ of the house's price P and borrows (1- $\square$ ) P at a monthly mortgage rate S (equal to the annual percentage rate divided by 12). With a conventional loan amortized over months, the monthly payment Y is

$$
\begin{equation*}
Y=\frac{S(1 \square \square) P}{\square 1+S} \tag{6}
\end{equation*}
$$

and the unpaid balance $\mathrm{B}_{\mathrm{t}}$ at the end of t months is

$$
\begin{equation*}
\mathrm{B}_{\mathrm{t}}=\frac{\mathrm{Y}}{\mathrm{~S}} \tag{7}
\end{equation*}
$$

If the homebuyer sells the house after $\mathrm{n}<\mathrm{m}$ months, the NPV is

$$
\begin{equation*}
N P V=\square \square P \square \square_{t=1}^{n} \frac{Y \square \square \mathrm{SB}_{t \mathrm{t} 1}}{(1+\mathrm{R})^{\mathrm{t}}}+\square_{\mathrm{t}=1}^{\mathrm{n}} \frac{\mathrm{X}_{\mathrm{t}}}{(1+\mathrm{R})^{\mathrm{t}}}+\frac{\mathrm{P}_{\mathrm{n}} \square \mathrm{~B}_{\mathrm{n}}}{(1+\mathrm{R})^{\mathrm{n}}} \tag{8}
\end{equation*}
$$

where R is now the monthly required rate of return. The first term is the down payment. The second term is the present value of the mortgage payments net of the tax savings from deducting
the interest-payments at a tax rate $\square$. The third term is the present value of the rent net of expenses; unlike the mortgage payments, these increase over time. The last term is the present value of the sale proceeds net of the loan balance.

If the homebuyer holds the house for keeps, the NPV is

$$
\begin{equation*}
N P V=\square \square P \square \square_{t=1}^{m} \frac{Y \square \square \text { SB }_{t \square 1}}{(1+R)^{t}}+\square_{t=1} \frac{X_{t}}{(1+R)^{t}} \tag{9}
\end{equation*}
$$

Some tedious algebra demonstrates that if the after tax required rate of return is equal to the after-tax mortgage rate, $R=(1-\square)$ S, Equations 8 and 9 simplify to the equivalent of Equation 5 :

$$
\mathrm{NPV}=\square \mathrm{P}+\frac{\mathrm{X}_{1}}{(1+\mathrm{R})^{1}}+\frac{\mathrm{X}_{3}}{(1+\mathrm{R})^{3}}+\ldots+\frac{\mathrm{P}_{\mathrm{n}}}{(1+\mathrm{R})^{\mathrm{n}}}
$$

As $n$ becomes indefinitely large, this NPV is equivalent to Equation 3 with $X_{0}=-P$. Thus the mortgage doesn't affect the NPV in the special case where the after-tax mortgage rate is equal to the after-tax required return.

Although the math is tedious, the logic is straightforward. A general principle that applies to all loans is that, at any point in time, the present value of all future payments made by the borrower, discounted at the loan rate, is equal to the loan's current unpaid balance. Consider month t with an unpaid balance $\mathrm{B}_{\mathrm{t}-1}$ from the preceding month. The payment Y includes interest $\mathrm{SB}_{\mathrm{t}-1}$ on the unpaid balance and reduces the unpaid balance to $\mathrm{B}_{\mathrm{t}}=\mathrm{B}_{\mathrm{t}-1}-\left(\mathrm{Y}-\mathrm{SB}_{\mathrm{t}-1}\right)$. If the loan is then paid off, the present value of the payments $Y+B_{t}$ is equal to the previous month's unpaid balance:

$$
\begin{aligned}
\frac{\mathrm{Y}+\mathrm{B}_{\mathrm{t}}}{1+\mathrm{S}} & =\frac{\mathrm{Y}+\mathrm{B}_{\mathrm{t} \square 1} \square\left(\mathrm{Y} \square \mathrm{SB}_{\mathrm{t} \square 1}\right)}{1+\mathrm{S}} \\
& =\mathrm{B}_{\mathrm{t} \square 1}
\end{aligned}
$$

This logic holds period after period, so that the last payment reduces the unpaid balance to zero.
Now consider the present value of the homebuyer's net after-tax cash flow discounted by an
after-tax required rate of return $R=(1-\square) S$ :

$$
\begin{aligned}
\frac{\mathrm{Y} \square \square \mathrm{SB}_{\mathrm{t} \square 1}+\mathrm{B}_{\mathrm{t}}}{1+\mathrm{R}} & =\frac{\mathrm{Y} \square \square \mathrm{SB}_{\mathrm{t} \square 1}+\mathrm{B}_{\mathrm{t} \square 1} \square\left(\mathrm{Y} \square \mathrm{SB}_{\mathrm{t} \square 1}\right)}{1+(1 \square \square) \mathrm{S}} \\
& =\frac{\mathrm{B}_{\mathrm{t} \square 1}+\mathrm{SB}_{\mathrm{t} \square 1} \square \square \mathrm{SB}_{\mathrm{t} \square 1}}{1+(1 \square \square) \mathrm{S}} \\
& =\mathrm{B}_{\mathrm{t} \square 1}
\end{aligned}
$$

The present value of the net after-tax cash flow is again equal to the previous period's unpaid balance. It follows mathematically that, just as with mortgage payments to a bank, the present value of the after-tax cash flow over all periods is equal to the initial loan balance:

$$
\square_{t=1}^{n} \frac{Y \square \square \mathrm{SB}_{\mathrm{t} \square \mathrm{l}}}{(1+(1 \square \square) S)^{\mathrm{t}}}+\frac{\mathrm{B}_{\mathrm{n}}}{(1+(1 \square \square) \mathrm{S})^{\mathrm{n}}}=(1 \square \square) \mathrm{P}
$$

If the after-tax mortgage rate is equal to the after-tax required return, leverage doesn't matter. Otherwise, we need to take the mortgage into account since the present value of the net cash flow associated with the mortgage will be larger or smaller than the amount borrowed depending on whether the after-tax required return is smaller or larger than the after-tax mortgage rate.

## Constant Growth

In some special cases, the model can be simplified. Suppose, for example, that the rent net of expenses increases at a rate $\mathrm{g}_{\mathrm{X}}$ and the house's price increases at a rate $\mathrm{g}_{\mathrm{p}}$. In this special case, the NPV equation is

$$
\begin{equation*}
N P V=\square \square P \square \square_{t=1}^{n} \frac{Y \square \square \operatorname{SB}_{t \square 1}}{(1+R)^{t}}+\square_{t=1}^{n} \frac{X_{1}\left(1+g_{x}\right)^{t \square 1}}{(1+R)^{t}}+\frac{P\left(1+g_{P}\right)^{n} \square B_{n}}{(1+R)^{n}} \tag{10}
\end{equation*}
$$

As before, the four terms are the present values of the down payment, mortgage payments net of the tax savings, rent net of expenses, and the sale proceeds net of the loan balance.

We can also simplify by assuming that the mortgage is an interest-only loan with no
maturation date:

$$
N P V=\square \square P \square \square_{t=1}^{n} \frac{(1 \square \square) S(1 \square \square) P}{(1+R)^{t}}+\square_{t=1}^{n} \frac{X_{1}\left(1+g_{x}\right)^{t \square 1}}{(1+R)^{t}}+\frac{P\left(1+g_{P}\right)^{n} \square(1 \square \square) P}{(1+R)^{n}}
$$

If the rent and price growth rates are less than the required return and house is held forever, this NPV equation simplifies to

$$
\begin{equation*}
\mathrm{NPV}=\square \square \mathrm{P} \square \frac{(1 \square \square) \mathrm{S}(1 \square \square) \mathrm{P}}{\mathrm{R}}+\frac{\mathrm{X}_{1}}{\mathrm{R} \square \mathrm{~g}_{\mathrm{x}}} \tag{11}
\end{equation*}
$$

Finally, we see that if the after-tax required return is equal to the after-tax required mortgage rate, $R=(1-\square) S$, the NPV is equivalent to Equation 2:

$$
\begin{equation*}
\mathrm{NPV}=\square \mathrm{P}+\frac{\mathrm{X}_{1}}{\mathrm{R} \square \mathrm{~g}_{\mathrm{x}}} \tag{12}
\end{equation*}
$$

The model generally doesn't simplify so nicely because: (a) a mortgage has a finite life; (b) mortgage payments build up equity; (c) the portion of the mortgage payment that is taxdeductible interest declines over time, and (d) the after-tax required return is not equal to the after-tax required mortgage rate. We can use a spreadsheet to handle this complexity.

## From NPV to PV

We will first use the simple model to illustrate how an NPV calculation can be used to determine the price one is willing to pay for a house. Equations 6 and 7 show that the mortgage payment and unpaid balance are both proportional to the initial loan balance:

$$
\begin{aligned}
Y & =[\mathrm{S}](1 \square \square) \mathrm{P} \\
\mathrm{~B}_{\mathrm{t}} & =\square_{\mathrm{t}}[\mathrm{~S}](1 \square \square) \mathrm{P}
\end{aligned}
$$

With these substitutions, the NPV given by Equation 10 is equal to 0 if

$$
\begin{equation*}
P=\frac{\square_{t=1}^{n} \frac{X_{1}\left(1+g_{x}\right)^{t \square 1}}{(1+R)^{t}}}{\square+(1 \square \square) \square_{t=1}^{n} \frac{[S] \square \square S \square_{t \square I}[S]}{(1+R)^{t}} \square \frac{\left(1+g_{P}\right)^{n} \square \square_{n}[S](1 \square \square)}{(1+R)^{n}}}=P \square_{-+}^{+}+\mathrm{X}_{1}, \mathrm{~g}_{x}, \mathrm{~g}_{P}, S, \mathrm{Q}, \mathrm{R}, \stackrel{+}{\square}, \stackrel{?}{\square}[ \tag{13}
\end{equation*}
$$

This is the reservation price-the highest price that this homebuyer would be willing to pay for this house. As shown, the reservation price is related positively to the net rent and the growth of rent and price, negatively to the mortgage rate and the required return, and positively to the tax rate. The fraction of the purchase that is financed with a downpayment has a positive effect on the reservation price if the after-tax required return is less than the after-tax mortgage rate and has a negative effect if the reverse is true.

## A General Analysis

Leveraged real estate can be valued by calculating the NPV of the cash flow. The guiding principle is to determine the after-tax cash flow each year. Don't be sidetracked by accounting labels. All we really care about is the dollars coming in and dollars going out. Cash is King!

As with stocks, we can use a spreadsheet to handle detailed cash-flow projections. We record each cash payment or receipt as it occurs so that we can take into account the time value of money. If we are sticklers for timing, we can use the exact dates on which mortgage payments are made, property taxes are paid, and so on. Because the projected cash flows are guesstimates, it is generally sufficient to work with monthly or annual approximations.

Another issue is whether to calculate the NPV for a given required return or to calculate the internal rate of return (IRR) that makes the NPV equal to zero. The IRR has the virtue of identifying a breakeven required return for which the investor is indifferent about the investment, but it also has several potential pitfalls, including the possibility of (a) an inverted NPV curve (with positive NPV for $\mathrm{R}>\mathrm{IRR}$ and negative NPV for $\mathrm{R}<\mathrm{IRR}$ ) if the cash flow is positive in
the early years and negative in later years; or (b) multiple IRRs if there is more than one sign change in the cash flow.

It is safest to calculate the NPV for a variety of required rates of return; however, the IRR can often be used safely because the cash flow is generally negative in the early years and positive in later years, with just one sign change. A situation that might make the IRR misleading is major remodeling expenses that give negative cash flows in the future and more than one sign change.

A free program for calculating the NPV and IRR for the buy-rent comparison is available at this web site: http://www.economics.pomona.edu/GarySmith/ivyplanner. Table 1 shows a summary spreadsheet for HW, a single 34-year-old college professor who lives in a Los Angeles suburb. She wants to compare buying versus renting for a 3-bedroom, 2-bath house with approximately 2,000 square feet of living space located in an attractive area with similar homes. The price of the house is $\$ 400,000$ and she will make an $\$ 80,000$ down payment. We assume that rents, housing prices, and most of her housing expenses will grow by 4\% a year. Although we used monthly data, Table 1 shows an annual summary for selected years.

1. The first column records the years.
2. The rent savings are what HW would have to pay to rent this house. We assume that the rent is initially $\$ 2,000$ a month and grows by $4 \%$ a year. One attractive feature of a house's implicit rental income is that it is an after-tax cash flow. If HW would pay $\$ 24,000$ a year in after-tax income to rent a house, then home ownership gives HW an extra $\$ 24,000$ in after-tax cash that she otherwise would pay in rent.
3. The third column records the mortgage payments. We assume a 30 -year $\$ 320,000$ mortgage at a fixed $6 \%$ interest rate. We report the total payments here and take into account the tax savings after determining the interest portion of the mortgage payment.
4. Annual property taxes are initially $\$ 4,000$ ( $1 \%$ of the acquisition cost) and grow by $2 \%$ a
year, as limited by California's Proposition 13.
5. Mortgage interest and property taxes are an itemized deduction with a tax saving equal to the amount paid multiplied by the HW's marginal tax rate. If HW pays state income taxes that are deductible from federal income taxes, her net marginal tax rate is equal to $\mathrm{m}=1-(1$ $-\mathrm{f})(1-\mathrm{s})$, where f is the marginal federal tax rate and s is the marginal state tax rate. HW's marginal tax rate $m$ is $35 \%$.
6. Column 6 encompasses utilities, insurance, maintenance and other expenses that HW will make if she buys instead of renting. These are generally not tax-deductible unless part of the home is used for business purposes. Her anticipated total is $\$ 8,520$ and is projected to grow by $4 \%$ a year. If any major remodeling expenses are anticipated, the cash outlays should be recorded in the year they will occur and any effect on the market value of the house should be recorded in column 8.
7. Column 7 is the net cash flow each year if the house is not sold. This net cash flow is the sum of the entries in columns 2-6.
8. For the sale price, we assume that the market value of the house increases at the same $4 \%$ rate as does the rental value, and that the brokerage commission and other expenses associated with the sale are equal to $8 \%$ of the sale price. Currently, capital gains up to $\$ 250,000$ for a single person and up to $\$ 500,000$ for a married couple filing jointly are not taxable; these limits will probably be increased in the future, though tax laws are always difficult to predict. We assume that HW's capital gain will not be taxed.
9. The mortgage balance is shown with a negative sign since it will be a cash outflow if the house is sold and the mortgage is paid off. The prepayment penalty, if any, should be included if the mortgage is paid off early. This mortgage had no prepayment penalty.
10. Column 10 is the NPV for an $8 \%$ after-tax required rate of return. If, for example, the house
is sold in year 5 , the NPV is calculated using the $\$ 80,000$ downpayment in year 0 , the net cash flow in column 7 for years $1-4$, and a net cash flow in year 5 equal to the sum of columns 7,8 , and 9 .
11. Column 11 uses the spreadsheet's IRR function to calculate the internal rate of return that sets the NPV equal to zero. Because these are after-tax cash flows, this is an after-tax IRR.

In our example, the net cash flow is negative for the first 7 years, but the homeowner is building up equity in an appreciating asset and the IRR is positive by the third year. Figure 1 shows the NPVs for 5-, 10-, and 20-year horizons using required returns ranging from 0 to $20 \%$. The NPVs increase with the holding period because (a) the mortgage payments are fixed while the net rental savings grow over time, causing the cash flow to go from negative to increasingly positive; and (b) the homeowner is building up equity in an appreciating asset. The IRRs are where the NPV curves cross the horizontal line at $\mathrm{NPV}=0$. HW should buy if her after-tax required rate of return is less than the IRR and rent otherwise. In the current financial environment, few investments look so promising.

Yet another way to analyze the data is to calculate her reservation price for a given horizon. Table 2 shows these results for required returns of 6,8 , and $10 \%$. The horizon of 10 years assumes that the price will appreciate at the same $4 \%$ rate as rents over this period. The infinite horizon ("for keeps") does not make any assumptions about the future price since it assumes that she will never sell the house.

## What a Difference a Model Makes

This example illustrates the weaknesses of simpler approaches. For instance, it is clearly misleading to ignore property taxes, utilities, insurance, and maintenance-which, on an after-tax basis, are approximately two-thirds the size of mortgage payments.

Even if these other expenses are taken into account, it misleading to compare the initial annual cost of home ownership with the initial annual rent. In our example, the annual cash flow is negative the first year, with the rent $\$ 3460$ less than the mortgage payments plus other expenses. Those who simply compare current rent with current housing expenses would conclude that renting is less expensive. But the rent saving grows over time, while the mortgage payments do not, and the mortgage payments build up equity in a property that is appreciating in value. Even with the $8 \%$ sales expense, HW can anticipate a positive NPV if she lives in the house for more than a few years.

More generally, the after-tax cash flows from buying a house are typically small or negative for the first few years, as the rental saving barely covers (or fails to cover) the costs of home ownership. As time passes, with rent growing and mortgage payments fixed, the after-tax cash flow becomes a substantial positive number. In addition, the homeowner's equity is growing, but can easily be swamped by substantial selling costs if the house is sold soon after purchase. These transaction costs underlie the generally sound advice that most people should not buy a house unless they plan to keep it for a while.

This cash-flow structure also creates a potentially fatal flaw for analyses that examine just a single horizon of, say, 3,5 , or 7 years. Suppose we look at a 5-year horizon and find that the NPV is negative. This doesn't necessarily mean that the house is a bad investment. Maybe it will be a good investment if we stay in the house for 8 years. Or maybe it won't. The only way to find out is to look at several horizons. Similarly, suppose we look at a 7 -year horizon and find that the NPV is positive. That doesn't necessarily mean that the house is a good investment if we stay in the house for only 3 years.

What about the house's $\mathrm{P} / \mathrm{E}$ ? In our example, the current $\mathrm{P} / \mathrm{E}$ is $\$ 400,000 / \$ 24,000=16.7$. Is that high or low? We can't tell unless we forecast the future costs and benefits of home
ownership and calculate the NPV using a required rate of return. What about the change in the P/E? This is even less informative. Housing prices happen to have increased much faster than rents in this particular area over the past several years, causing the housing $\mathrm{P} / \mathrm{E}$ to increase substantially. But an increased P/E doesn't necessarily mean this house is a bad investment. With our plausible assumptions, this house looks like an excellent investment.

## Sensitivity Analysis

As with any intrinsic-value model, investors should not be dismayed by the fact that they cannot provide exact values for the future cash flow. We don't need to know the values to the last penny. The way to handle imperfect knowledge is to try a range of values. More generally, it is a good idea to do a sensitivity analysis to see whether the NPV is reasonably robust or depends critically on certain key assumptions. We will look at three examples.

First consider mortgage rates of $8 \%$ and $10 \%$, in addition to the $6 \%$ base case. Figure 2 shows the NPVs for a 10-year horizon and Figure 3 shows the IRRs for horizons up to 30 years. The effects of mortgage rates on the NPVs are very strong because the financial market conditions that increase interest rates also increase the prospective buyer's required rate of return. Suppose, for example, that the mortgage rate and required return both increase from $6 \%$ to $8 \%$ and then $10 \%$. The NPV falls from $\$ 62,246$ (point A) to $\$ 9,307$ (point B) and then $-\$ 34,444$ (point C). Two conclusions are apparent: (a) higher interest rates make buying less appealing, and (b) a cash-flow approach to valuing a house should take mortgage rates into account.

Figure 4 shows the IRRs for $0 \%, 4 \%$, and $8 \%$ growth rates of rent, housing prices, and various expenses. The growth rate is clearly a crucial parameter. The purchase will not be financially rewarding unless there will be some growth in rents and housing prices.

The price is also a crucial parameter. As with any investment, there is surely some price at which a house is too expensive. At $\$ 400,000$, this house looks like a good investment if rents and
prices increase at plausible rates. Figure 5 shows the IRRs if the price of this house were instead $\$ 600,000$ or $\$ 800,000$. At $\$ 800,000$, the house loses a lot of its luster.

## The Palo Alto Bubble

What about those modest Palo Alto houses selling for $\$ 2$ million? Similar houses could be rented for around $\$ 3,000$ a month. We will increase the annual insurance cost to $\$ 3500$, but not change utilities and maintenance. Table 3 shows the spreadsheet. The mortgage payments and property taxes swamp the rent savings, giving a negative annual cash flow of between $\$ 60,000$ and $\$ 70,000$ over the entire 30 -year period. The only positive aspect is the assumption that the house's price will rise by 4\% a year, which gives a substantial capital gain when the house is sold. Even so, the IRR is never higher than $5 \%$ and the NPV at an $8 \%$ required return is always a large negative amount.

Table 4 shows that the reservation prices for $4 \%$ price growth, $3 \%$ price growth, and buying for keeps are much lower than the $\$ 2$ million market price. Such prices may be justified if the homebuyer is willing to settle for a modest rate of return. Otherwise, it looks like a bubble fueled by expectations that prices will continue to rise rapidly.

## References

Alexander, Barbara. "Borrowing to Save," http://www.moneydots.com/column.html, 2003.
Conti, Peter, and Finkel, David . Making Big Money Investing in Real Estate, Chicago: Dearborn Financial Publishing, 2002.

Detweiler, John H. and Radigan, Ronald E., "Computer-Assisted Real Estate Appraisal: A tool for the Practicing Appraiser," The Appraisal Journal, 1996, 64: 91-102.

Detweiler, John H. and Radigan, Ronald E., "Computer-Assisted Real Estate Appraisal: A tool for the Practicing Appraiser (2)," The Appraisal Journal, 1999, 67: 280-286.

Eldred, Gary W. Value Investing in Real Estate, New York: John Wiley \& Sons, 2002, 267-268.
Feldman, Amy. "A P/E for Your Home?," Money, July 2003, 107-108
Goodman, Jordan E. Everyone’s Money Book, Chicago: Dearborn Trade, 2001.
Isakson, Hans R., "The Review of Real Estate Appraisals Using Multiple Regression Analysis," The Journal of Real Estate Research, 1998, 15: 177-190.

Joint Center for Housing Studies of Harvard University, "The State of the Nation's Housing," 2002.

Kapoor, Jack R., Dlabay, Les R., and Robert J. Hughes. Personal Finance, New York: Irwin/McGraw-Hill, 1999, 268-270.

Keown, Arthur J. Personal Finance, Upper Saddle River, New Jersey: Prentice-Hall, 1998, 253256.

Miller, Ted. Kiplinger's Practical Guide to Your Money, Kiplinger, 2002.
Nguen, N. and A. Cripps. "Predicting Housing Value: A comparison of Multiple Regression Analysis and Artificial Neural Networks, Journal of Real Estate Research, 2001, 22: 313336.

Orman, Suze. The Road to Wealth, New York: Riverhead, 2001.

Quinn, Jane Bryant. Making the Most of Your Money, New York: Simon \& Schuster, 1997, 1000-1001.

Ramaglia, Judith A., and MacDonald, Diane B. Personal Finance, Cincinnati, Ohio: SouthWestern, 1999, 250-251.
de Roos, Dolf, Real Estate Riches, New York: Warner Books, 2001.
Winger, Bernard J., and Fresca, Ralph R. Personal Finance, Upper Saddle River, New Jersey: Prentice-Hall, 2000, 375-377.

Table 1 Claremont Spreadsheet

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Rent | Mortgage | Property | Tax | Other | Net Cash | Net Sales | Mortgage | NPV | IRR |
| Year Savings | Payments | Taxes | Savings | Expenses | Flow | Price | Balance | R $=8 \%$ | $(\%)$ |  |
| 1 | 24000 | -23023 | -4000 | 8083 | -8520 | -3460 | 382720 | -316070 | -21773 | -22.8 |
| 2 | 24960 | -23023 | -4080 | 8026 | -8861 | -2978 | 398029 | -311896 | -12513 | -0.2 |
| 3 | 25958 | -23023 | -4162 | 7964 | -9215 | -2477 | 413950 | -307469 | -4143 | 6.3 |
| 4 | 26997 | -23023 | -4245 | 7898 | -9584 | -1957 | 430508 | -302767 | 3412 | 9.0 |
| 5 | 28077 | -23023 | -4330 | 7826 | -9967 | -1417 | 447728 | -297774 | 10219 | 10.3 |
| 10 | 34159 | -23023 | -4780 | 7374 | -12127 | 1604 | 544730 | -267794 | 35035 | 11.4 |
| 15 | 41560 | -23023 | -5278 | 6726 | -14754 | 5232 | 662747 | -227356 | 48617 | 11.0 |
| 20 | 50564 | -23023 | -5827 | 5809 | -17950 | 9573 | 806333 | -172811 | 55033 | 10.5 |
| 25 | 61519 | -23023 | -6434 | 4526 | -21839 | 14749 | 981028 | -99239 | 56957 | 10.1 |
| 30 | 74848 | -23023 | -7103 | 2742 | -26571 | 20893 | 1193570 | 0 | 56135 | 9.8 |

Table 2 Reservation Prices for Claremont House

| Required Return | Sell after 10 years | Buy for keeps |
| :---: | :---: | :---: |
|  | $4 \%$ price growth |  |
| $6 \%$ | $\$ 734,000$ | $\$ 780,000$ |
| $8 \%$ | $\$ 557,000$ | $\$ 471,000$ |
| $10 \%$ | $\$ 451,000$ | $\$ 364,000$ |

## Table 3 Palo Alto Spreadsheet

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Rent | Mortgage | Property | Tax | Other | Net Cash | Net Sales | Mortgage | NPV | IRR |
| Year Savings | Payments | Taxes | Savings | Expenses | Flow | Price | Balance | R $=8 \%$ | $(\%)$ |  |
| 1 | 36000 | -115114 | -20000 | 40413 | -11300 | -70001 | 1913600 | -1580352 | -159348 | -37.2 |
| 2 | 37440 | -115114 | -20400 | 40129 | -11752 | -69697 | 1990144 | -1559492 | -161533 | -13.1 |
| 3 | 38938 | -115114 | -20808 | 39821 | -12222 | -69385 | 2069750 | -1537345 | -166237 | -5.5 |
| 4 | 40495 | -115114 | -21224 | 39489 | -12711 | -69065 | 2152540 | -1513833 | -173170 | -1.9 |
| 5 | 42115 | -115114 | -21649 | 39130 | -13219 | -68737 | 2238641 | -1488870 | -182069 | 0.1 |
| 10 | 51239 | -115114 | -23902 | 36871 | -16083 | -66989 | 2723649 | -1338972 | -248399 | 3.5 |
| 15 | 62340 | -115114 | -26390 | 33630 | -19568 | -65101 | 3313736 | -1136782 | -355983 | 4.3 |
| 20 | 75847 | -115114 | -29136 | 29046 | -23807 | -63165 | 4031667 | -864057 | -430881 | 4.5 |
| 25 | 92279 | -115114 | -32169 | 22628 | -28965 | -61341 | 4905139 | -496193 | -524958 | 4.6 |
| 30 | 112271 | -115114 | -35517 | 13710 | -35241 | -59890 | 5967851 | 0 | -613748 | 4.7 |

## Table 4 Reservation Prices for Palo Alto House

| Required Return | Sell after 10 years |  | Buy for keeps |
| :---: | :---: | :---: | :---: |
|  | $4 \%$ price growth | $3 \%$ price growth |  |
| $6 \%$ | $\$ 1,171,000$ | $\$ 855,000$ | $\$ 1,245,000$ |
| $8 \%$ | $\$ 888,000$ | $\$ 709,000$ | $\$ 751,000$ |
| $10 \%$ | $\$ 719,000$ | $\$ 608,000$ | $\$ 580,000$ |



Figure 1 NPVs for Different Horizons


Figure 2 NPVs for a 10-year Horizon and Mortgage Rates of $6 \%, 8 \%$, and $10 \%$


Figure 3 Annual After-Tax IRR for Mortgage Rates of 4\%, $6 \%$, and $8 \%$


Figure 4 Annual After-Tax IRR for Growth Rates of $0 \%, 4 \%$, and $8 \%$


Figure 5 Annual After-Tax IRR for Prices of $\$ 400,000, \$ 600,000$, and $\$ 800,000$

